ML-TDR-64-298

MECHANICS OF ADHESIVE BONDED LAP-TYPE JOINTS: SURVEY AND REVIEW

TECHNICAL DOCUMENTARY REPORT No. ML-TDR-64-298

DECEMBER 1964

AIR FORCE MATERIALS LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO ,45433

Project No. 7381, Task No. 738103

(Prepared under 33(657)-63-358 by the
Forest Products Laboratory, Forest Service, U. S. Department of
Agriculture, Madison, Wisconsin; Dieter Kutscha, Author)

NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Qualified requesters may obtain copies of this report from the Defense Documentation Center (DDC), (formerly ASTIA), Cameron Station, Bldg. 5, 5010 Duke Street, Alexandria, Virginia, 22314.

This report has been released to the Office of Technical Services, U.S. Department of Commerce, Washington 25, D. C., for sale to the general public.

Copies of this report should not be returned to the Research and Technology Division, Wright-Patterson Air Force Base, Ohio, unless return is required by security considerations, contractual obligations, or notice on a specific document.

600 - January 1965 - 448-24-559

FOREWORD

This report was prepared by the Forest Products Laboratory. Work here reported was sponsored by the AF Material Laboratory under USAF Contract No. 33(657)63-358. This contract is carried under Project No. 7381, "Material Application," Task No. 738103, "Materials Information Development, Collection and Processing." It is administered under the direction of the AF Materials Laboratory, Research and Technology Division, Mr. T. J. Reinhart, Jr., project engineer.

Material tested may not have been developed or intended by the manufacturer for the conditions to which it was subjected. Performance is therefore not necessarily indicative of its utility under less stringent conditions or for other applications.

This technical documentary report has been reviewed and is approved.

D. A. Shinn

D. A. SHINN
Chief, Materials Information Branch
Materials Applications Division
AF Materials Laboratory

ABSTRACT

This report presents a critical review made in the field of the design of adhesive bonded, lap-type joints. The study included a comprehensive survey and analysis of the literature pertaining to the theoretical and experimental analysis of lap joints, mechanical properties of adhesive film in joints, failure criteria for joints, and empirical methods of joint design. Based on the results of this survey, some recommendations are given for further research in this area.

TABLE OF CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (Cont.)

LIST OF SYMBOLS

A, B, C,	=	constants
E	=	modulus of elasticity of adherends
E_a	=	modulus of elasticity of adhesive
G	=	modulus of rigidity of adherends
G_a	=	modulus of rigidity of adhesive
P	=	applied load
T	=	applied torque
b	=	joint width
d_1, d_2	=	tube radii
L	=	length of overlap
n	=	Stress concentration factor = T_m/T_a
t_1, t_2	=	adherend thicknesses
t_a	=	adhesive film thickness
u, v, w	=	strains along coordinate axes
x, y, z	=	coordinate axes
δ	=	shear deformation
Δ	=	joint factor
ϵ	=	tensile or compressive strain
γ_a	=	average shear strain
γ_m	=	maximum shear strain

λ	=	a constant
$\sigma_x, \sigma_y, \sigma_z$	=	coordinate longitudinal strains
τ	=	shear stress
τ_a	=	average shear stress
τ_m	=	maximum shear stress
μ	=	Poisson's ratio - adherend

I. INTRODUCTION

A. Purpose

The purpose of this study was to provide a critical review of the field of design of adhesive bonded lap-type joints. This was to be accomplished through an analysis of the literature and a review of current design practice in use by some of the major manufacturers of adhesive-bonded structures. Based on the results of this survey, recommendations are made for further research in this area.

The development of high strength structural metal-bonding adhesives has made possible two major advances in airframe structures. These are: (1) higher strength-to-weight ratios and (2) longer fatigue life. These advances have resulted in such aircraft as the B-58 "Hustler" Bomber with its adhesive-bonded sandwich construction and the all metal adhesive-bonded helicopter rotor blade. The design of these high-performance structures has been based on fairly simple empirical methods with a strong reliance on the pragmatic approach of building a structure or model, and testing it to determine whether the structure meets the criteria for non-failure; then accepting the design if the criteria are met. Only in scattered instances has it been necessary to undertake any extensive analysis work to solve a certain problem. This is no longer the situation.

Today the problem is twofold. First, the adhesive user has available almost the entire spectrum of possible mechanical properties in adhesives. This has resulted from continuing advances in organic polymer chemistry and a better understanding of the mechanical behavior of polymers. Second, the requirements for adhesives are becoming more stringent. Longer fatigue life in a structure is always desirable. High temperature service and strength requirements are such that conventional organic adhesives are no longer applicable and it has become necessary to turn to the "metallic" adhesives or brazes. This has resulted in the development of brazed honeycomb-sandwich construction.

In order to adequately handle these problems, a better understanding of the design of an adhesive joint has become necessary to better utilize the materials that are available and to make possible the rational design of more advanced structures. As the report progresses, it will be seen that the major emphasis in the past has been on design of metal-bonded structures. It should be remembered that the general principles involved are just as applicable to the design of plastic or wooden joints, and it is with these basic principles that we are concerned.

Manuscript released by author June, 1964 for publication as an RTD Technical Documentary Report.

To discuss this subject properly, it is necessary to introduce the concept of an adhesive joint itself as a structural assembly. It is assumed that both the adhesive and adherends are individual structural components of the assembly, and that each has its own inherent mechanical properties. This means the adhesive has a purpose other than to just "stick" the adherends together in some manner. The joint is considered as a composite of materials having widely varying properties, and the properties of the adhesive can have a marked effect on the overall performance of the joint.

The ultimate objective is to develop a design method for this bonded construction, based on the principles of mechanics of materials and rational engineering design so that joint behavior can be predicted. Consider now what is meant by the phrases "mechanics of bonded joints" and "rational engineering design." The following comments are based on the text by Seely and Smith (57).[1]

The function of a structural adhesive joint is to resist an external load. If the joint fails to function properly it will undergo structural damage or failure. This structural damage could be actual fracture of the structure, excessive elastic deformation, or excessive inelastic flow. The criteria for what constitutes structural failure will depend on the performance requirements of the joint. The fundamental problem in mechanics of materials and joints is to obtain some relationship between the loads applied to the joint and a parameter that will adequately describe the criteria for structural failure. For example, if a joint is considered to have failed when its elastic deflection reaches a certain limit, a relationship is necessary between applied load and elastic deflection. Similarly, relationships may be necessary between joint strength and applied load, or creep and applied load. The most common criteria for structural failure of lap-type joints is actual fracture of the joint. The phenomena which lead to structural failure are termed the mode of failure.

Based on the above considerations, a procedure for the rational engineering design of a joint can be developed. Commonly four steps are considered:

1. Determine the mode of failure for the structural joint.

For a given combination of adherend and adhesive a decision must be made as to what the mode of failure of the joint would be if the applied loads become large enough to cause failure. There are six main theories of failure, namely:

---

[1] Underlined numbers in parentheses refer to Literature Cited under References at the end of this report.

a. Maximum Principle Stress Theory
b. Maximum Shear Stress Theory
c. Maximum Strain Theory
d. Total Strain Energy Theory
e. Strain Energy of Distortion Theory
f. Octahedral Shear Stress Theory

The decision as to which theory would properly determine the mode of failure could be based on past experience or some form of experimental evidence.

2. Determine a relationship between the applied load and a parameter (stress, strain, strain energy, etc.) that will describe the failure of the joint.

This expression would include the joint dimensions. Usually it is convenient to obtain relationships for the principal stresses as a function of the applied load (the shear stress distribution) since from these parameters other quantities (strain energy) can be calculated that may describe the structural failure.

Stress distributions maybe obtained either analytically or experimentally. A mathematical analysis is based on: (a) The principles of mechanics, (b) assumptions concerning the magnitude of the deformations involved, and (c) a knowledge of the mechanical properties of the materials in the structure. An experimental analysis is based on techniques of experimental stress a analysis, or on experimental analogy methods that solve equations developed in the mathematical analysis in a mechanical manner.

3. Determine the maximum properties of the materials in the structure which must be exceeded at structural failure.

Assuming that the joint will fail when the principal stress in the adhesive perpendicular to the plane of joint (tensile stress) reaches a level sufficient to cause fracture, it is necessary to know the strength of the adhesive under tension in this direction. Similarly, it may be necessary to know the maximum shear stress, maximum elastic deformation, etc.

4. Determine allowable stress values from the maximum stresses obtained in step 3.

It is at this point that factors of safety are decided upon for factors such as long and short term loading, fatigue loading, special environmental conditions and other special considerations. This step is usually based on experience, engineering judgment, and legal or commercial specifications.

For the purpose of this report, the work conducted under steps (1) and (2) in relation to adhesives was of the greatest interest. Based on the concept of a rational design the work reviewed fell logically into the following categories.

(1) Theoretical analyses of the stress distribution in adhesive-bonded lap joints.
(2) Experimental stress analyses of lap joints.
(3) Mechanical properties of adhesive films in joints.
(4) Failure criteria for lap joints.
(5) Empirical methods of joint design.

Unfortunately it is not always possible to completely carry out steps (1) and (2) for a rational design of a structure, and it becomes necessary to resort to an empirical approach based on either testing models of the structure in question or actually testing the structure. Testing is continued with modifications to the structure until the desired performance is obtained. This approach can be time consuming and costly, but it may be the only approach possible. In the past, this pragmatic approach has been extensively used for adhesive-bonded structures and has given satisfactory results. A review of these methods is given in the section, "Empirical Methods of Joint Design."

One assumption made which concerns this concept of adhesive-joint design, is that the adhesive forms an adequate chemical bond with the adherend in all instances. This means that the adhesives considered for structural application are actually capable of forming a load-bearing structure. With this assumption one can neglect all the problems of manufacture of the joint.

This study is restricted to lap-type joints for two reasons:

(1) Lap-type joints are the most common type occurring in adhesive-bonded assemblies. Almost all bonded joints can be simplified to a lap joint for analysis. The only real exception to this is the true butt joint.

(2) Lap joints are the primary type of joint used for determining the strength of adhesive joints.

There is good reason for this reliance on a lap-type configuration. If the joint is properly designed, the primary stress on the adhesive will be a shear stress, under which adhesives exhibit their greatest strength. Adhesive joints are notoriously weak in "peel" and are also weak under tensile loads applied normal to the plane of the joint. It is therefore of primary importance that one have an understanding of the mechanics of the lap joint, in order that any high peel stresses can be eliminated.

It is also important to thoroughly understand the routine lap-joint test, so one can properly interpret the results. As with many so-called simple mechanical tests of materials, they are simple because the specimens are easily made, and not because they provide any type of simple or pure stress condition. It will be seen later that lap joints subject the adhesive to a complex combination of shear and tensile stresses.

B. Scope

The literature survey portion of this study was based on a review of the following:

(1) The Engineering Index (1914 to date)
(2) Applied Mechanics Reviews (1948 to date)
(3) NACA-NASA Index of Technical Publications (1915-1960)
(4) Subject Index to Unclassified ASTIA Documents (Documents 1-75,000)
(5) ASTIA Technical Abstract Bulletin

An effort was made to make the survey as complete as possible, especially in the government literature. In all instances, a wide list of subject titles was searched.

The survey was not a 100 percent search of the literature but it is believed all the documents of major importance were uncovered.

In the literature survey, the author was guided by the reviews of DeBruyne and Houwink (50), Perry (55), Benson (8), and Sneddon (60). These reviews are all recommended as excellent references in this area of mechanics of adhesive joints. Sneddon (60) particularly gives a good review of the major papers.

The information obtained from the literature survey is presented to develop the general progress in the area of mechanics of joints, and to provide a comprehensive guide for the reader. An annotated bibliography of applicable literature surveyed is included as a part of each main section of the report (Sections II-VI). An effort was made to provide sufficient comment and information on each reference so the reader could decide whether he desires to search the original paper further. This is especially true for some of the more obscure references which may be difficult to obtain. A complete alphabetical listing of all literature surveyed is listed under References at the end of this report.

The Industry Survey portion of the study was conducted in the following manner. Initially a letter of inquiry with a questionnaire attached was sent to 49 members of the Aerospace Industries Association, representing companies most likely to be concerned with adhesive bonding. Nineteen responded to the initial questionnaire, and eight of these appeared to have sufficient information to warrant a visit and personal discussion. The eight companies visited are listed in Appendix B.

The information obtained from these visits pertained primarily to the general philosophy of the use of structural adhesives, particular design problems, and general practices of adhesive bonding, quality control, and use. Reports that were available are included in the literature survey.

Although no specific report is given of these visits, they served as a background or base upon which this report is written. The influence of the visits and personal discussions is reflected throughout the report as a central theme for the comments and reviews presented. It should be pointed out that the comments and reviews express the author's own opinion unless otherwise indicated.

C. Description of a Lap Joint

The questions now arise as to why should one obtain a nonuniform stress distribution in a simple lap-type joint? and what properties of the joint will affect the stress distribution in the joint?

A typical adhesive-bonded lap joint is illustrated in Figure 1. According to DeBruyne (16) there are two major reasons why one would expect a non-uniform stress distribution in the joint, namely: (1) differential straining and (2) eccentric loading.

Note in Figure 2 the deflection under load in a grid scribed on a joint prior to loading. In one adherend the tensile strain will vary from a maximum at one end of the joint to a minimum at the other end. In the other adherend the same distribution occurs only with the position of maximum and minimum strain reversed when compared to the first adherend. The adhesive film is exaggerated for clarity, but note how it must absorb the differential straining of the two adherends. From this simple diagram it is evident that the shear strains are maximum at the ends of the adhesive in the joint and fall to a minimum at the center. Several researchers have analyzed this model since it is the simpler of the two cases.

The other reason for a non-uniform stress distribution is illustrated in Figure 3. Note that the line of force drawn through the unloaded joint does not

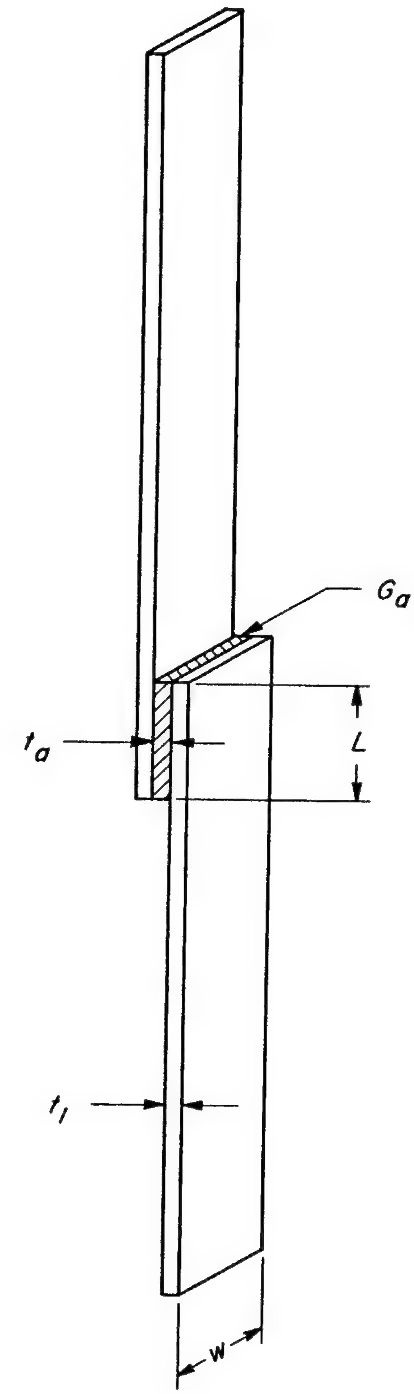


Figure 1. --Typical Adhesive Bonded Lap-Type Joint

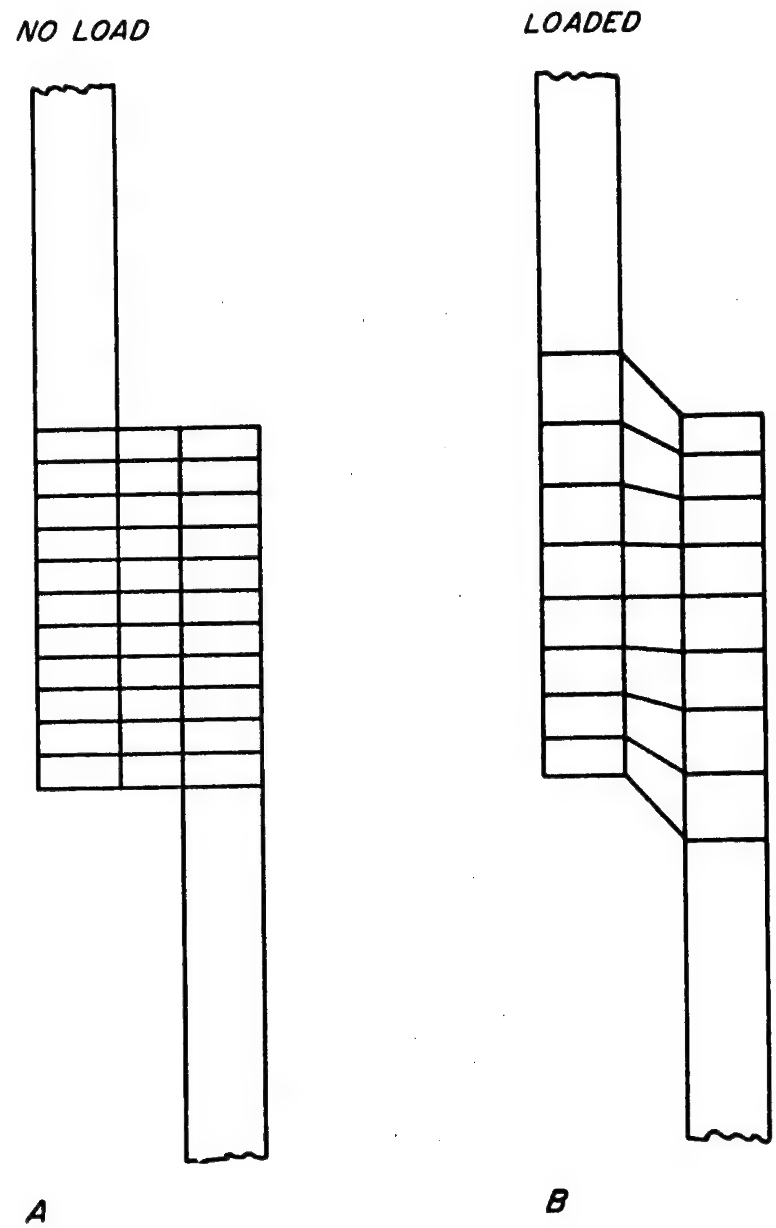


Figure 2. --Differential Straining in a Lap Joint Showing the Change in a Reference Grid from: A, Before Loading to B, After Loading.

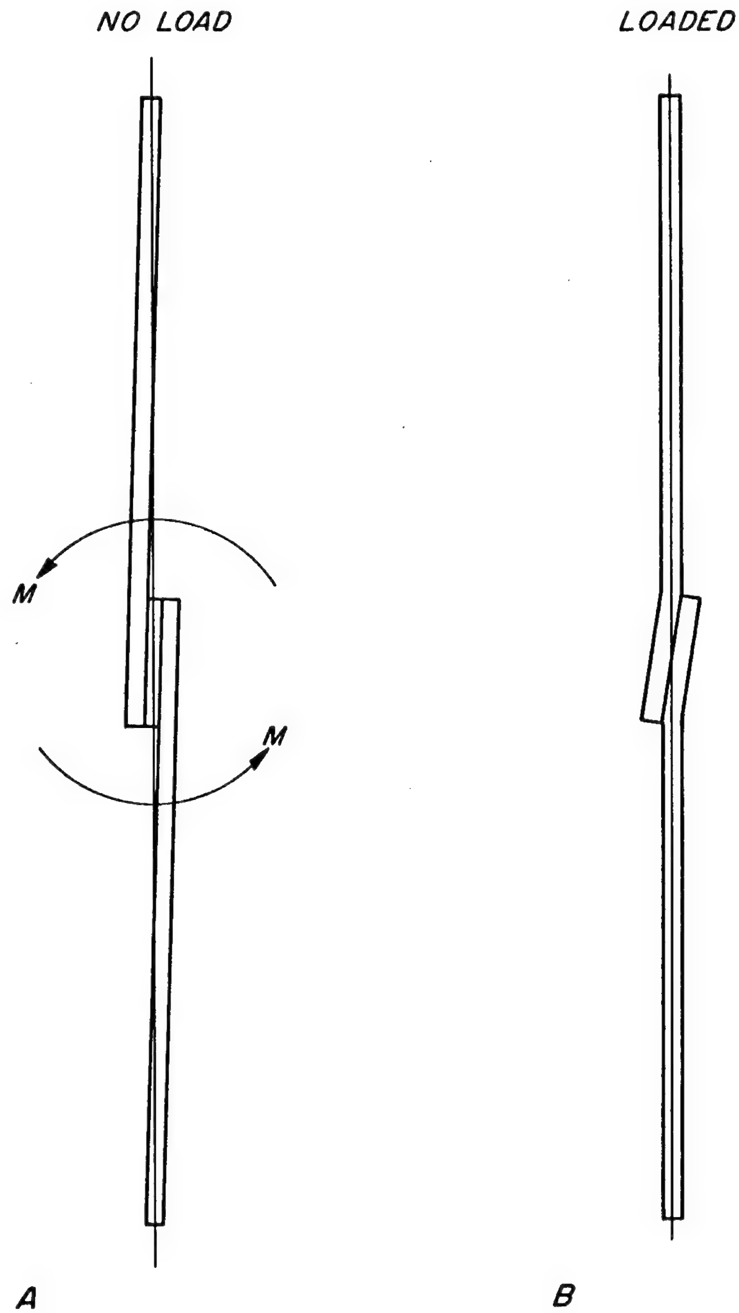


Figure 3.--Bending Moment in a Lap Joint That is Due to the Eccentric Load on the Joint Area: A, Line of Force Through the Joint Before Loading; B, Line of Force Through the Joint After Loading.

pass through the center of the adhesive film. At the ends of the bonded area the line of force is offset by a slight eccentricity. This eccentric loading induces a bending moment which tends to rotate the joint until the line of force passes directly through the center of the adherend. This bending moment decreases as the joint rotates, but adds to the complex stress distribution that the adhesive must absorb. This bending condition is much more difficult to describe mathematically and has received less attention than the simpler differential straining.

The problem is to adequately describe mathematically the stress distribution in the adherends and, more important, in the adhesive since the adhesive is usually weaker than the adherends under these complex load conditions.

The two classes of independent variables that affect the distribution are joint geometry and mechanical properties.

The factors included in joint geometry are:

1. Length of overlap.
2. Adhesive thickness.
3. Adherend thickness.

Mechanical properties are:

1. Adhesive moduli of rigidity and of elasticity.
2. Adherend moduli of rigidity and of elasticity.

An acceptable analysis would include all these factors and the applied load. If possible it should go beyond the elastic, isotropic materials and include anisotropic materials with visco-elastic behavior.

II. THEORETICAL ANALYSIS OF BONDED JOINTS

A. Problems of Analysis

The description of any physical system by some form of mathematical formulation is based on the following operations:

1. Choice of a model or idealization. Certain assumptions are made initially to make the problem tractable to mathematical formulation.

2. Mathematical description of the idealized physical model, usually a differential equation.

3. Solution of the mathematical equations.

4. Interpretation of the results obtained from the mathematical model to determine their correlation with the physical system and whether they are physically meaningful.

In the analysis of the literature we are primarily concerned with reviewing the analyses presented in the light of operations one, two, and four. Step three is the realm of the mathematician and, in all instances, the solutions to the differential equations were straightforward.

The most important step to be considered is number one, the choice of a model or idealization. It is this operation that determines how well the results obtained from the mathematics will fit the physical system. If the assumptions made and boundary conditions chosen are too unreasonable or simplify the physical system markedly, the results predicted by the analysis cannot be expected to describe the physical system adequately. As previously mentioned, one of the assumptions usually made is that no bending occurs in the lap joint. Another assumption is that all materials in the structure behave elastically.

The lap joint is a statically indeterminate structure. It is not possible to simply analyze the structure based on the equations of static equilibrium alone. It is necessary to use the requirements of static equilibrium as well as the requirements of geometry or continuity. It is possible to solve statically indeterminate structures using methods from classical structural mechanics or modern numerical methods. Both of these methods have been applied to the lap-joint problem.

It should be pointed out that the problem of stress distribution in adhesives is not unique or even relatively new to mechanics. The problem of shear loading or shear transfer from one body to another is almost classic to the field of strength of materials. The problem occurs whenever loads are transferred by shear across an interface. This includes almost all types of adhesive joints; bolted, nailed, and riveted joints; shrink fit joints; shear transfer through individual bolts, nails and rivets; shear transfer from fibers to resin matrix in reinforced fiber glass materials; shear transfer in dams; shear transfer in reinforced concrete beams from the bars to the concrete; and many others. Thus, the general principles involved in analyzing adhesive joints have application to all the above situations.

The one overall assumption that appears repeatedly throughout all the analyses is that of elastic or Hooke's law behavior for all the materials in the structure. For metal-bonded joints, where the adhesive will fail before the metal, this assumption is suitable for the metal adherends. For organic adhesives or organic adherends this assumption is not as reasonable. Organic materials almost universally exhibit viscoelastic character. The stress is not a linear function of the strain but is dependent on the rate of strain application, the strain level, and time. The stress is also temperature dependent.

These various factors become very important when joints are loaded to destruction, as in routine adhesive evaluation using lap joints. Due to the stress concentrations in the joint, certain areas of the adhesive would be expected to go beyond the elastic limit well before the joint might fail; or if the joint is not loaded to failure, some irreversible flow would occur and affecting the later behavior of the joint. This point will be considered further under section IV on mechanical properties of adhesive films.

B. Linearized Small Deflection Theory of Elasticity

The majority of the analyses presented are based on the linearized, small deflection theory of elasticity. The equations from the theory of elasticity are modified by assuming that only small deflections or strains occur, and all the second order terms in the equations of strain can be neglected. This can be further simplified to the equations of engineering elasticity, in which stresses are replaced by loads and support conditions replace boundary and compatibility conditions, by the requirement that the deformation be continuous. This can also lead to a degenerate Hooke's law equation where Poisson's ratio is neglected. This is the procedure followed in most of the analyses presented.

When choosing idealized models, assumptions are made such as: (1) the adherends act as beams on an elastic foundation, (2) the adherends act as thin plates in bending, or (3) the adhesive acts as an infinite plate, so that only two dimensional effects need to be considered. Each assumes behavior according to a specific type of engineering structure which implies that other assumptions are generally made when attempting to analyze one of these types of units. All of the assumptions cannot be adequately discussed in this survey beyond stating the type of structure assumed.

C. Major Assumptions Used in the Analyses

There are several major assumptions made that are common to many of the analyses. These are listed and briefly discussed below. The extent to which they detract from the analysis is difficult to assess, because only in a few instances are they supported by any experimental evidence, and also because the analysis may have met the situation for which it was originally intended.

1. Elastic Behavior of Materials. - This assumption was common to all the analyses investigated. Since organic materials are commonly viscoelastic, this assumption would limit most analyses to joints under low stress. In only one report (26) was any attempt made to include plastic behavior of the adhesive. To use elastic analyses for joints loaded to destruction is a questionable procedure.

2. Joint Bending. - It was usually assumed in most analyses that the joint (double-lap joint configuration) was restrained from bending. This condition is less complicated to analyze and actually does occur in most structures utilizing adhesives. The geometry of the structure is such that the bonded area cannot rotate and normal stresses should be at a minimum. The more complicated bending condition, as occurs in the simple lap-joint test, is more difficult to analyze. The assumption of no bending is a reasonable one to make depending on the structure involved.

3. Adhesive Thickness. - In some analyses it was assumed that the adhesive was infinitely thin. In this instance the influence of the adhesive on the properties of the joint was neglected and the joint model corresponded to a tension member with an offset. The stress distribution obtained was that caused by the offset in the line of force, rather than any discontinuity through the member such as an adhesive bond line.

4. Adhesive Properties. - The mechanical properties (G_a and E_a) of the adhesive varied from the same order of magnitude as that of the adherends to very much softer. This would cover the range of E = 300 psi to 10×10^6 psi. In most analyses the adhesive was generally in the order of 100,000 psi. The properties of the adhesive were considered to be uniform throughout the joint.

5. Stress Distribution in the Adhesive. - The stress distribution was always assumed to be uniform through the thickness of the adhesive from one adherend to the other. Several experimental studies have shown this assumption to be not valid, especially at the ends of the overlap (48, 37).

The adhesive was usually assumed to be in a condition of plane stress, neglecting the stresses in the width direction of the joint. Assuming the adhesive to be in a condition of plane strain would appear to be more realistic.

6. Air-Adhesive Interface. - In most instances no stipulation was made concerning the shape of the extruded adhesive or braze at the end of the overlap (the squeeze-out). In many applications this cannot be controlled and has a very irregular shape.

D. Differential Straining

A single lap joint undergoing loading due to differential straining of the adherends was the most common analysis found in the literature. This was the analysis carried out by Arnovlevic (7), Volkersen (64), Wann and Sherwin (65), Goldenburg (25), Broding (9), Lubkin (42), Misztal (47), Albert (1), and Anonymous (6).

The problem was generally formulated in the following manner.

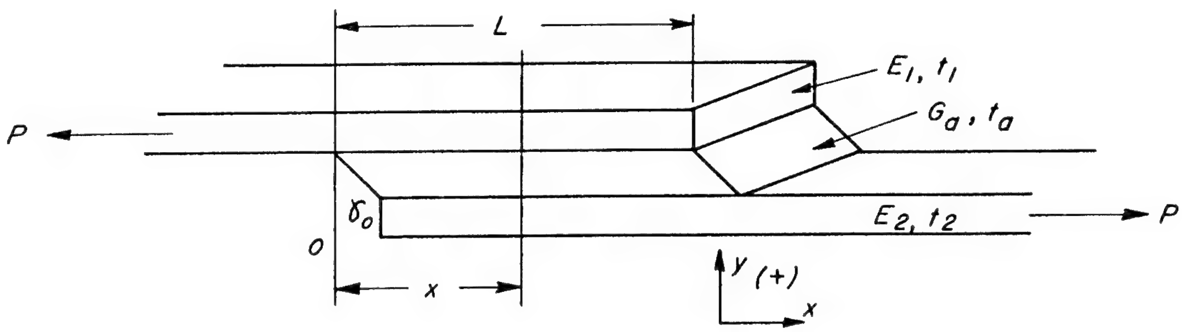


Figure 4.--Differential Straining in a Bonded Lap Joint.

$$\gamma_x = \gamma_o + \int_o^x \epsilon_{2_x} dx - \int_o^x \epsilon_{1_x} dx \tag{1}$$

Thru proper substitution, collection of terms and then differentiation, the differential equation

$$\frac{d\gamma_x^2}{dx^2} - \lambda^2 d\gamma_x = o \tag{2}$$

is obtained, where

$$\lambda^2 = \frac{2\,G_a}{E\,t} \tag{3}$$

when both adherends have the same thickness and modulus of elasticity.

This differential equation has the general solution of the form

$$\gamma_x = A \sinh(\lambda x) + B \cosh(\lambda x) \tag{4}$$

After determining A and B, solving for the maximum γ_x at $x = L$, assuming that $\gamma = T/G$ and non-dimensionalizing the expression by dividing by the average shear stress, one obtains for the maximum stress concentration factor the expression

$$n = \sqrt{\Delta/2}\left[\frac{1 + \cosh\sqrt{2\Delta}}{\sinh\sqrt{2\Delta}}\right] \tag{5}$$

or

$$n = \sqrt{\Delta/2}\ \coth\sqrt{\Delta/2}, \tag{6}$$

where

$$\Delta = \frac{G_a L^2}{E_1 t_1 t_a} \tag{7}$$

This expression assumes elastic behavior of all materials and a uniform adhesive. In this instance, each adherend is undergoing a parabolic shear strain distribution along its length with the maximum in each adherend opposite to the other. This nonuniform distribution must be absorbed by the adhesive.

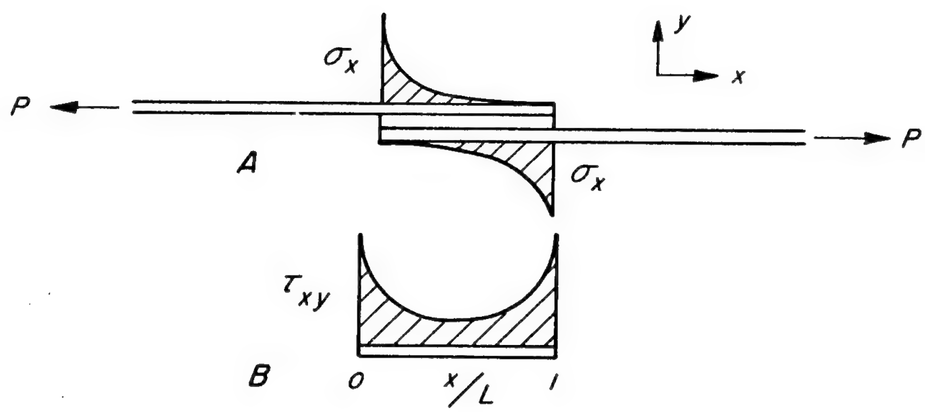


Figure 5.--Stress Distribution in a Simple Lap Joint Showing: A, Adherend Stress Distribution; B, Adhesive Stress Distribution.

Note that the maximum stress concentration factor (n) at the end of the length of overlap is a direct function of λ which is related to G_a and t_a. Therefore, by reducing the adhesive shear modulus or increasing the film thickness the maximum shear stress can be reduced.

An extensive series of calculations showing the effect of various joint parameters on the maximum stress concentration was presented by Lubkin (42). The calculations are for tubular lap joints but are applicable to the plate lap joint without bending.

Albert (1) modified the analysis to include the effect of tapering the adherends in thickness throughout the length of overlap. Essentially this changes the parabolic normal stress distribution in the adherends to a uniform distribution and subjects the adhesive to a uniform shear distribution. However, it was found that the adherend must be tapered to zero thickness at the end of the overlap to accomplish this. The point of zero thickness is very difficult to obtain in practice.

In one instance, Anonymous (6), this type of analysis was actually used to solve a practical problem. Based on the analysis it was possible to choose the shear modulus and thickness for any adhesive that could absorb the differential strains generated in the interlayer between two dissimilar materials in a cylindrical shell. In this particular instance a very low modulus material (G_a = 100-300 psi) was necessary.

Filon (22), Inglis (32), and Niskanen (51), representing some of the newest and some of the oldest work, have used the methods of theory of elasticity to analyze this problem. Filon and Inglis describe the stress distribution in a body subject to pure shear which would be similar to the load situation of an adhesive in a joint. Niskanen assumes a state of plane stress and describes the stress distribution in the adherends of a double lap-type joint. Both isotropic and anisotropic adherends were analyzed and for the isotropics it was believed a non-uniform peaked distribution was obtained, but for the anisotropics almost a uniform shear distribution occurred. This latter point is important to the testing of lap-type wood joints.

E. Joint Under Bending

It is more difficult to analyze a lap joint unrestrained to bending and subsequently there are fewer papers available. Account must be taken of the bending moment caused by the offset in the line of force through the joint as it is initially loaded in tension. This moment is a maximum initially and then decreases as the applied load rotates the joint. At maximum rotation the line of force will pass through the center of the adherends. In this rotated condition the adherends are behaving as beams in bending. The face of the

adherend bonded to the adhesive will be subjected to a tensile stress that is superimposed on the tensile stress already present due to the applied load on the joint.

In the adhesive the stress condition is complicated by the presence of high normal or peel stresses superimposed on the tensile stress already present due to the applied load on the joint.

In the adhesive the stress condition is complicated by the presence of high normal or peel stresses superimposed on the usual stresses due to differential straining.

The work of Goland and Reissner (24) is the classic paper in this field. In their paper, the adherends were assumed to behave as cylindrical plates in bending. For a complete discussion of this work the reader is strongly encouraged to read Sneddon (60). Two cases were discussed by Goland and Reissner; in one the presence of the adhesive is neglected, and in the other the adhesive is assumed relatively flexible compared to the adherends. In the latter case, the normal and shear stress distribution were determined and their relationship to the degree of bending indicated.

Plantema (56) combined the differential straining condition with the Goland and Reissner analysis.

The method of Volkersen (64) was used to determine the loads applied at the edge of the overlap, and then the analysis of Goland and Reissner was used to determine the stress distribution in the adherends throughout the bonded area.

Cornell (15) modified the Goland and Reissner analysis further by assuming the adherends behaved as simple beams in bending unaffected by the presence of the adhesive. Equations were obtained for the beam (adherend) deflections and the shear and normal stress distributions along the adherend-adhesive interface. The stress distribution in the adhesive was obtained by assuming it was the same as that along the interface. The analysis showed the importance of the end of the overlap on the stress distribution.

Sherrer (59) attempted an analysis of a joint with an elastic adhesive by assuming the adherends to behave as plates in bending (plane stress) and the adhesive to be in a state of plane strain. Difficulty was encountered in obtaining meaningful stress distribution.

Ito (33) considered a lap joint in which the overlap area was assumed to behave as a simple beam in bending subjected to several different load conditions. The presence of any adhesive was neglected and expressions were obtained for the deflections of the neutral axis. The work does point out the importance of loading lap joints reproducibly when comparison strength tests are conducted.

In a final paper Hahn and Fouser (30) determined the stress distribution in the adherends outside the bonded area by assuming them to behave as cylindrically bent plates. The results were analogous to the other analyses of the stress distribution within the bonded area, in that the stress distribution reaches a maximum at the edge of the overlap.

F. Matrix Structural Analysis

The application of matrix structural analysis methods to the lap joint problem is the most recent advance in this area. Presently the only two reports available concerning this topic are those of Lobbett and Robb (39) and Goodwin (26).

This analysis method is based on the idealization of a lap joint by a network model of bar and shear panel elements. By choosing a proper size or fineness of grid, using mechanical properties of the bars and shear panels to coincide with adherends and adhesive in the respective areas, it is possible to completely idealize a lap joint and subject it to any desired loading situation. Equations are written for the behavior of each element, and the equations must be then solved simultaneously for the entire network. Through the use of matrix algebra the solution to these equations is placed in a language ideal for the digital computer. With the use of the computer the problem becomes tractable and a large number of calculations can be made concerning the stress distributions, which could not even be attempted with any other analysis method.

Lobbett and Robb (39) initially considered the application of this technique to the lap joint problem and were able to show its superiority over other available methods. Goodwin (26) has since used the method to investigate an extensive number of joint parameters and their effect on the stress distribution in a lap-type joint. The parameters included: Length of overlap, braze (adhesive) modulus, film thickness, joint support (bending), adherend taper, and air-braze interface radius. Only the elastic stress distributions have been investigated but the method can be extended to include plastic material behavior.

As a result of this study a theory was developed for the strength of a lap joint. The theory was based on the assumption that the joint will fracture under load when the braze has undergone a certain amount of plastic yielding at the ends of the overlap, and that when this plastic zone reaches a certain size the joint will fail. The size of this plastic area is a material property of the braze and must be known. The theory gave excellent agreement with static strengths of actual joints.

Although this study was concerned with brased joints, meaning the adhesive was very stiff and had properties similar to that of the adherends, the techniques applied and information obtained are also applicable to adhesive joints involving more flexible materials.

To summarize the analysis work it can be stated there are a variety of analysis methods available, all of which are concerned with elastic behavior, and they establish the importance of the length of overlap and adhesive properties on stress distribution. A complete explanation of the behavior at the end of overlap is not available, particularly as influenced by the local effects of irregular shaped air-adhesive interfaces.

The analysis method that could readily yield more information is that based on the matrix structural techniques. Further effort should be made to extend this technique to more conventional lap joints with organic adhesives.

G. Annotated Bibliography

Filnon, L.N.G. (22)

> On an approximate solution for the bending of a beam of rectangular cross-section under any system of load, with special reference to points of concentrated or discontinuous loading. Part III. Solution for a beam under a symmetric normal forces: Special case of two opposite concentrated loads not in the same vertical straight line. Royal Society of London - Philosophical Transactions, A 201, p. 63, 1903.

This paper presents an approximate solution to the stress and strain distribution in a rectangular six-sided elastic body under any system of load. Beginning with the basic equations from the theory of elasticity, the problem is simplified to two dimensions by assuming that two of the faces remain plane; consequently the normal forces perpendicular to these faces are zero and the shear stresses vanish at these plane boundaries.

These approximations are approached by two conditions: that of bending of a thick plate, and bending of a flat beam where its height is large compared to width.

The importance of this paper lies in the solution to the problem of a beam stressed by two concentrated loads not in the same plane (Fig. 6).

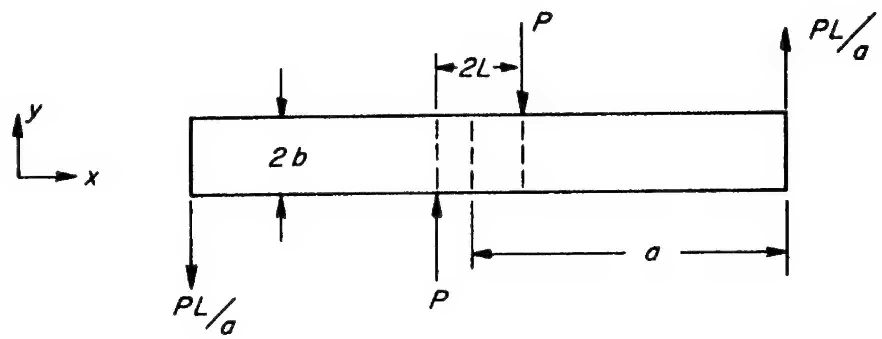


Figure 6.--A Beam Stressed by Two Concentrated Loads Simplified to Two Dimensions.

Note that in the central section of the beam when L becomes small compared to b we approach the condition of an adhesive joint under pure shear, where adhesive and adherend have the same mechanical properties. Although this condition is not achieved in most joints, either the pure shear or the similar mechanical properties, it is of interest since Filon computed the shear stress distribution over the section as a function of the length or film thickness of the adhesive. Filon points out this condition approximates that of two plates held together by a rectangular rivet and the plates being loaded in their own plane.

The elastic stress distributions obtained are shown in Figure 7. Note that the highest stress concentrations are obtained for the thinnest adhesive and even for an infinitely thick adhesive a stress concentration of 1.5 was found.

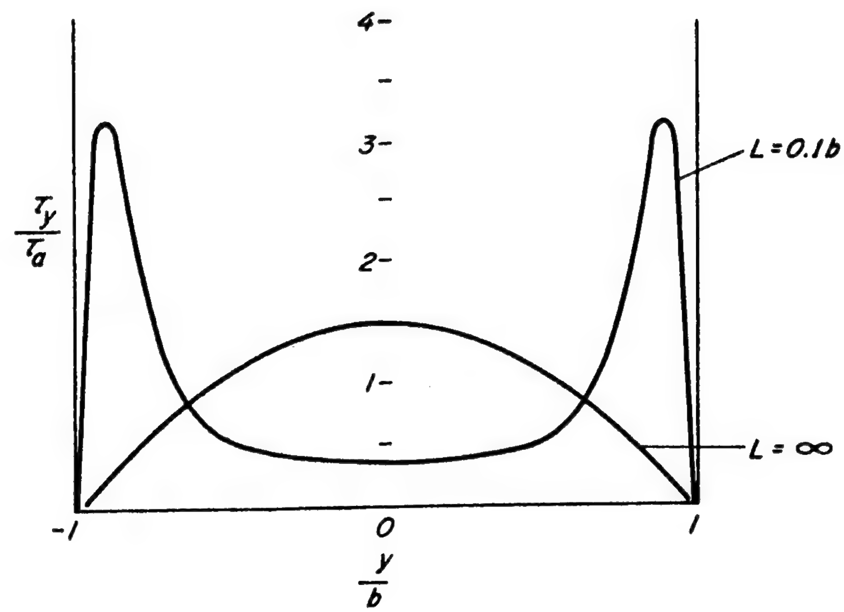


Figure 7.--Elastic Stress Distributions in a Beam Stressed by Two Concentrated Loads Simplified to Two Dimensions.

Arnovlevic, Ivan (7)

Das Verteilungsgasetz der Haftspannung bei Axial Beanspruchten Verbunstaben, Zeitschrift fur Architektur und Ingeniorwesen, Hannover, 2:414-418. 1909.

This paper pertains to two concentric bonded bars in which the inner bar is axially loaded and the "bonding" could be due to (a) friction, (b) shrink fit, or (c) adhesion with an adhesive (Fig. 8).

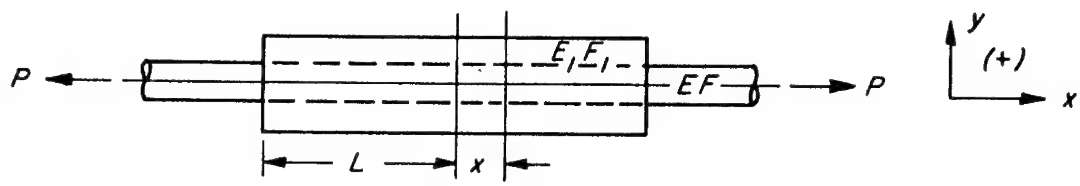


Figure 8.--Two Concentric Bonded Bars in which the Inner Bar is Axially Loaded, and the Bonding Due to Friction, Shrink Fit, or Adhesion With an Adhesive.

The materials are assumed elastic and the "adhesive" has zero thickness. The two conditions analyzed are: (1) inner bar continuous and (2) inner bar with a discontinuity at the center.

The differential equation for the stress distribution at the interface between the bars is the same in both instances.

$$\frac{d\sigma_x{}^2}{dx} = a^2\sigma_x - b \qquad (8)$$

where

σ_x = average normal force parallel to length of the bar at position (x).

x = position along the bonded area measured from the center of the area.

a^2, b = constants.

and $$\tau_x = \frac{F}{u}\,\frac{d\sigma_x}{dx} \qquad (9)$$

where

τ_x = shear stress at x

F = cross sectional area of inner bar

σ_x = normal stress parallel to bar axis at x

u = circumference of inner bar

The general solution to the differential equation (8) has the form

$$\sigma_x = C_1 e^{ax} + C_2 e^{-ax} + b/a^2 \tag{10}$$

Solving for the constants C_1, C_2 using the boundary conditions

(1) $x = 0, \quad \dfrac{d\sigma_x}{dx} = 0$ and $\sigma_x = 0$

(2) $x = L, \quad \sigma_x = S$ where $S = P/F$

one obtains for Case (1)

$$\sigma_x = \frac{S}{1 + \epsilon\ \phi}\left[\epsilon\ \phi + \frac{\cosh ax}{\cosh aL}\right] \tag{11}$$

$$\tau_x = \frac{Pa}{u(1 + \epsilon\ \phi)}\ \frac{\sinh ax}{\cosh aL} \tag{12}$$

Using the boundary conditions

(1) $x = 0, \quad \sigma_x = 0$

(2) $x = L, \quad \sigma_x = S$

one obtains for Case (2)

$$\sigma_x = \frac{S}{1 + \epsilon\ \phi}\left[\epsilon\ \phi + \operatorname{csch} aL \left\{\sinh ax = \epsilon\ \phi \sinh a(L - x)\right\}\right] \tag{13}$$

$$\tau_x = \frac{Pa.\ \operatorname{csch} aL}{u\ (1 + \epsilon\ \phi)}\left[\cosh ax - \epsilon\ \phi \cosh a(L - x)\right] \tag{14}$$

where

P = load on inner bar

L = one half the length of overlap

$\epsilon = E/E^1$

$\phi = F/F^1$

S = stress in inner bar, outside the overlap

F, F^1 = cross sectional area of inner and outer bar

E, E^1 = modulus of elasticity - inner and outer bar

T = "shear modulus" of the interface, "adhesive"

u = circumference of the inner bar

and $$a^2 = T \frac{FE + F^1E^1}{FE.F^1E^1} \tag{15}$$

$$b = \frac{Ts}{F^1E^1} \tag{16}$$

This analysis represents the lap joint without bending and assumes a very thin adhesive with the stress distribution across the adhesive film to be uniform. The form of the differential equation obtained is of interest since it is found to occur again in later papers.

Inglis, C. E. (32)

Stress Distribution in a Rectangular Plate Having Two Opposing Edges Sheared in Opposite Directions. Proc. Royal Soc. (London) Series A, 103 pp. 598-610, 1923.

Inglis presents an analytical investigation of a rectangular plate being sheared along two opposing edges, the same problem considered by Coker (14), Andrade (2), and Filon (22). The results were compared with the photoelastic investigation of Coker.

The problem is formulated as one in plane stress using the theory of elasticity. A single stress function V is assumed to satisfy the following:

$$\nabla V^4 = 0 \tag{17}$$

$$\delta^2 V/\delta_y 2 = \sigma_x \tag{18}$$

$$\delta^2 V/\delta_x 2 = \sigma_y \tag{19}$$

$$-\delta^2 V/_{\delta x \delta y} = \tau_{xy} \tag{20}$$

where

$$V = - A/n \; x \; e^{-nx} \sin ny \tag{21}$$

is utilized for edges AD and BC,

and $V = -B n^2 \sin n x \left(\frac{u-1}{u+1} \cosh n - n \sinh n\right) \sinh ny + ny \cosh n \cosh ny$ (22)

is used for edges AB and CD.

The problem is treated in two parts using the principle of superposition.

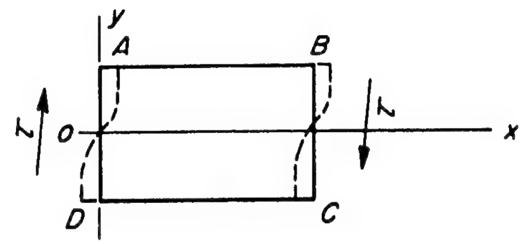


Figure 9.--Rectangular Plate Being Sheared Along Two Opposing Edges.

(1) A uniform shear stress is applied along AD and BC with AB and CD held absolutely rigid. The resulting ϵ_x along AB is determined and a shear stress necessary to overcome this is applied to AB. This induces nonuniformity and a shear strain back on AD.

By successive approximations a proper function is found which gives a uniform shear stress along AD with zero ϵ_x along AB. The normal force τ_y along AB to balance the τ_{xy} along AD is determined.

The above τ_{xy} is applied along BC.

(2) A uniform τ_{xy} is subtracted from all four edges of the rectangle. This cancels the shear stress along AD and BC and gives a uniform shear along the other two edges, which is the desired load condition.

The shear stress distribution was then determined along the center line of the plate and found to be in excellent agreement with photoelastic results of Coker.

An important observation by Inglis was that the normal forces found necessary to balance the shear loads were four times as high as the uniform applied shear stress. He points out that, if tested to destruction, the plate would probably fail by tearing at a corner. This point is of importance in adhesive applications because of the low tensile strength of adhesives.

Volkersen, O. (64)

Die Nietkraftverteilung in Zugbeanspruchten Nietverbindungen mit Konstanten Laschonquerschnitten. Luftfahrtforschung 15 :41-47, 1938.

This article is the one usually mentioned as the earliest paper on theoretical analyses of lap joints. It is extensively discussed by Mylonas and DeBruyne (50), Perry (55), and Sneddon (60).

Volkersen considers the elastic stress distribution in a lap joint in which only differential straining is taking place. The presence of peeling or normal forces is neglected.

The differential equation describing the strain distribution has the form

$$\frac{d\gamma_x{}^2}{dx^2} - \lambda^2 \frac{d\gamma_x}{dx} = 0 \tag{23}$$

$$\lambda^2 = \frac{G_a t_1 + t_2}{E\, t_1 t_2} \tag{24}$$

for the joint with adherends of unequal thickness.

The solution of this equation leads to a relationship for the stress concentration factor (n):

where $$n = \frac{\gamma_{xy}}{\gamma_a} \tag{25}$$

and since the stress is proportional to the strain

$$n = \frac{\tau_{xy}}{\tau_a} \tag{26}$$

and

$$n = \frac{(\Delta/W)^{1/2}}{\sinh(\Delta/W)^{1/2}} \left[(w-1)\cosh\left((\Delta . W)^{1/2} \cdot \frac{x}{L} \right) + \cosh\left((\Delta . w)^{1/2} \left[1 - \frac{x}{L} \right] \right) \right] \tag{27}$$

$$\Delta = \frac{G_a L^2}{E \cdot t_2} \tag{28}$$

$$w = \frac{t_1 + t_2}{t_1} \tag{29}$$

$$\gamma_a = \frac{P}{b. L. G} \tag{30}$$

$$\tau_a = \frac{P}{b. L} \tag{31}$$

When both adherends have the same modulus and the same thickness

$$w = \frac{t_1 + t_2}{t_1} = 2, \quad \Delta = \frac{G_a L^2}{E\, t_1} \tag{32}$$

and the maximum shear stress concentration is desired, $x = 0$, equation (27) can be simplified to

$$n = \sqrt{\Delta/2} \left[\frac{1 + \cosh \sqrt{2\Delta}}{\sinh \sqrt{2\Delta}} \right] \tag{33}$$

where $$\frac{1 + \cosh \sqrt{2\Delta}}{\sinh \sqrt{2\Delta}} = \coth \frac{\sqrt{2}}{2} \tag{34}$$

therefore $$n = \sqrt{\Delta/2} \quad \coth \sqrt{\Delta/2} \tag{35}$$

and $$n = f(\Delta) \tag{36}$$

The important assumptions in this analysis were:

1. All materials in the joint behave elastically.

2. There are no bending moments and consequently no peel stress in the joint. .

3. The stress distribution across the adhesive film is a constant.

4. The properties of the adhesive film do not vary with a change in film thickness.

Goland M. and Reissner, E.(24)

The Stresses in Cemented Joints. Journal Applied Mechanics, 11(1):A17-27, 1944.

This classic study of the stresses in bonded lap-type joints has been reviewed and discussed so thoroughly by Mylonas and DeBruyne (50), Perry (55), and Sneddon (60) that only a brief review will be presented. The review by Sneddon (60) is highly recommended. Of most important to this study are the major assumptions used in developing the analysis.

An analysis was presented of the shear and normal stress distribution throughout the adhesive of a simple overlap-type joint. All materials were assumed to behave elastically, the stress distribution across the adhesive film was assumed to be constant, and the adhesive was assumed to be subjected to a condition of plane strain in the plane of the joint.

The general approach to the problem was as follows:

First the stress distribution was determined in the adherends just outside the bonded area. This provided a description of the loads being applied to the edge of the joint. The stress distribution was obtained using the theory of small bending for thin cylindrically-bent plates. Two special conditions were then investigated for the bonded area; in one, the adhesive layer was assumed to be infinitely thin and relatively inflexible so it could be considered as having no effect on the stress distribution in the adjoining adherends. In the other, the adhesive was assumed to have an effect on the bonded members. It was this second condition which provided an analysis of a metal-bonded lap-type joint.

In this second case the adherends were treated like cylindrically bent plates separated by a system of infinitesimal elastic coil-springs. This analysis lead to a differential equation similar to that for the deflection of a beam on an elastic foundation. In order to have this model describe the joint adequately the following criteria were used:

$$\frac{t_1}{E_1} \leq \frac{1}{10} \frac{t_a}{E_a} \quad \text{or} \quad \frac{t_1}{G_1} \leq \frac{1}{10} \frac{t_a}{G_a} \tag{37}$$

The following expressions were obtained for the shear and normal stress in the adhesive as a function of position in the length of overlap:

$$\tau_x = -\frac{P\,t_1}{4L} \left\{ \frac{\beta L}{t_1} (1 + 3K) \frac{\cosh \frac{\beta L}{t} \cdot \frac{X}{}}{\sinh \frac{\beta L}{t}} + 3(1-K) \right\} \tag{38}$$

where $\beta^2 = 8 \dfrac{G_a t_1}{E_1 t_a}$ (39)

$$\sigma_{y_x} = \frac{P t^2}{L^2 \nabla} \left[(R_2 \lambda^2 \frac{K}{2} + \lambda K^1 \cosh \lambda \cos \lambda) \cosh\lambda \frac{x}{L} \cos \lambda \frac{x}{L} \right.$$

$$\left. + (R_1 \lambda^2 \frac{K}{2} + \lambda K^1 \sinh \lambda \sin \lambda) \sinh\lambda \frac{x}{L} \sin\lambda \frac{x}{L} \right] \quad (40)$$

where $\lambda = \gamma \dfrac{L}{t_1}$ $\gamma^4 = 6 \dfrac{E_a}{E_1} \cdot \dfrac{t_1}{t_a}$ (41)

$$R_1 = \cosh \lambda \sin \lambda + \sinh \lambda \cos \lambda \quad (42)$$

$$R_2 = \sinh \lambda ;\cos \lambda - \cosh \lambda \sin \lambda \quad (43)$$

$$\nabla = \frac{1}{2} (\sinh 2 \lambda + \sin 2 \lambda) \quad (44)$$

$$K = \frac{2M_o}{pt^2}, \quad \text{and } K^1 \frac{V_o c}{pt^2} \quad (45)$$

The expressions involving M_o and V_o are related to the earlier part of the analysis where the joint edge loads were determined.

The importance of this paper lies in its approach to the problem of bending in the lap-type joint and the evaluation of the high normal stresses which lead to the peeling type failure so common in lap-joint testing. The major objection to the analysis is the assumption that the adhesive behaves elastically. If this is not a valid assumption for the adhesive the stress distribution will change very rapidly as the material creeps or behaves inelastically.

Dietz, A.G.H., Grinsfelder, H., and Reissner, E. (19)

Glue Line Stresses in Laminated Wood. ASME Transactions 68, pp. 329-335, May 1946.

A discussion and analytical expressions are given for the shear and normal stress distribution in the glue line of two flat boards bonded together over their entire width. Two conditions are discussed:

1. Two boards of equal thickness were bonded together and one member allowed to expand or contract while the other was held rigid. The adhesive was loaded only in shear.

2. Two boards of equal thickness bonded together were subjected to a parabolic stress distribution across their thickness. There was no shear stress in the glue line and only the normal stress was considered.

The expressions given for the maximum stresses were obtained from strain energy considerations with the materials behaving elastically. The adhesive was assumed to be infinitely thin and its properties were not included in the analysis. The complete analysis, with all the assumptions made, was not included.

Plantema, F. J. (56)

De Schuifspanning in een Lijmnaad. The Shear Stress in a Glue Joint. National Luchtvaart Laboratorium, Amsterdam. Report MI 181., 1947.

The shear stress distribution was determined for a simple lap joint loaded in tension parallel to the joint for two conditions; (1) a restrained joint without bending, and (2) a joint with bending.

For the joint without bending, a review of the analysis used by Volkersen (64) was used. For the joint with bending, a combination of the Volkersen, Goland, and Reissner analyses was used. Using the form given by Mylonas and DeBruyne (50), this led to the expression

$$\frac{\tau_m}{{}_{\iota}\tau_a} = \left[1/2\ \Delta\ (1 + 3K)\right]^{1/2} \coth \left[1/2\ (1 + 3K)\right]^{1/2} \tag{46}$$

where $$\Delta^2 = \frac{G_a L^2}{E_1\ t_1 t_a} \tag{47}$$

and $$1/K = 1 + 2\sqrt{2}\ \tanh\left\{ \left[\frac{3}{2}\ (1 - \mu^2)\right]^{1/2} \frac{L}{2\ t_1} \left[\frac{P}{E_1}\right]^{1/2} \right\} \tag{48}$$

The major assumption was that all materials behave elastically.

Goldenburg, D. (25)

Distribution of Shear Stresses in Bonded Joints. Chance Vought Aircraft Report 7441, Feb. 1948.

An elastic analysis is presented for several types of bonded joints including a simple lap joint with unequal adherends. The analysis used by Wan and Sherwin (65) for a lap joint with equal adherends was generalized to a joint with unequal adherends. The analysis assumed the joint to be loaded in shear only without bending.

The following expression was obtained:

$$\frac{\tau_m}{\tau_a} = \Delta \coth \Delta + \frac{t_2 - t_1}{t_2 + t_1} \Delta^2 \qquad \frac{1}{\Delta \coth \Delta} \tag{49}$$

in which

$$\Delta^2 = \frac{G_a \; L^2}{E_1 t_a \cdot \dfrac{t_1 t_2}{t_1 + t_2}} \tag{50}$$

Broding, W. C. (9)

Analyses for Mechanical and Bonded Joints in Metalite. Chance Vought Aircraft Engineering Dept., Report No. 7588, June 1952.

The design of Metalite sandwich panel details used to transfer loads from one panel to another is discussed. It was assumed that load transfer occurs through shear between overlapped panel faces and between panel and core. The shear stress distribution was determined using the Volkersen-type analysis.

Cornell, R. W. (15)

Determination of Stresses in Cemented Lap Joints. Journal of Applied Mechanics 20, No. 3:pp. 355-364, Sept. 1953.

An analysis is presented for the stress distribution in a lap-type joint consisting of a thin tab strip brazed to a thicker base bar. The composite was loaded in bending. Brittle lacquer and photoelastic stress analysis techniques were used to experimentally determine the stress distribution and these results were compared with those obtained analytically. The mechanical properties of the braze were assumed to be one half those of the adherends.

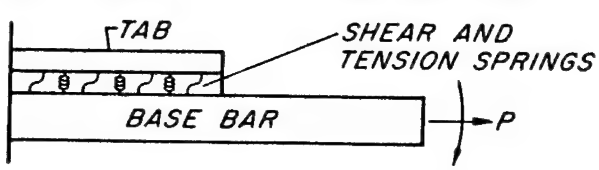


Figure 10.--A Cemented Lap-Type Joint Consisting of a Thin Tab Brazed to a Thicker Base Bar.

The analysis was based on a lap joint model in which the adherends were assumed to behave as individual simple elastic beams in bending, and the braze (adhesive) to behave as a thin layer of infinitely small elastic shear and tension springs. The beam stiffness of the braze was assumed to be negligible. The model was expressed by a tenth order differential equation of the general form.

$$D^2 (D^2 - A^2) (D^4 - 2 BD^2 + C^2) V = 0 \qquad (51)$$

Equations were obtained for the beam (adherend) deflections and for the σ_x, σ_y, T_{xy} distributions along the braze-adherend interface.

In the experimental program the brittle lacquer technique was used to determine the effects of adherend thickness, braze thickness, and braze modulus on the maximum principal stress in the base bar. Specimens investigated were 2 to 4 times their normal size. The photoelastic study was carried out on a composite model consisting of two different plastics with a modulus of elasticity ratio of 2 to 1. The models were four times normal size. The base bar was 0.25 inch, the tab 0.04 inch and the braze 0.01 inch in thickness.

The results of the analytical study showed the strong influence of the end of the overlap on the stress distribution; also, that the stress distribution became more uniform at the end of the overlap as the braze modulus decreased or the braze thickness increased. It was further shown that the maximum stresses increased as the tab thickness increased (became more rigid). It was found that the angle the maximum principal stress makes with the base bar changed from 0° to 90° as the braze thickness decreased from 0.01 inch to 0.001 inch.

It was generally found that the brittle lacquer and photoelastic studies checked the results obtained analytically.

Niskanen, E. (51)

On the Distribution of Shear Stress in a Glued Specimen of Isotropic or Anisotropic Material. The State Institute for Technical Research, Finland, Julkaisur 30 Publikation, 1955.

Niskanen developed an analysis for a double lap joint consisting of either isotropic or anisotropic adherends and subjected to a compressive load. The joint is shown in the Figure 11.

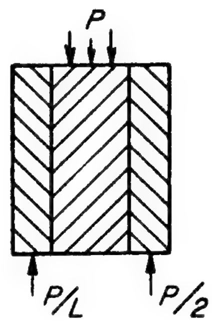


Figure 11.--A Double Lap Joint Consisting of Either Isotropic or Anisotropic Adherends and Subjected to a Compressive Loading.

This specimen is commonly used to determine the shear strength of bonded wood assemblies.

It was assumed that the materials behaved linear elastically and that a state of plane stress prevailed.

The differential equation for plane stress was used:

$$\frac{\delta^4 \phi}{\delta y^4} + 2\frac{\delta^4 \phi}{\delta y^2 \delta z^2} + \frac{\delta^4 \phi}{\delta z^4} = 0 \tag{52}$$

where ϕ is the Airy stress function.

The form for ϕ chosen was

$$\phi = -\frac{P}{4} z^2 + \sum_{n=1}^{\infty} \frac{1}{a_n^2} (A_n \sinh a_n y + a_n y B_n \cosh a_n y) \cos a_n \tag{53}$$

where $a_n = \frac{n\pi}{a}$.

The distribution for σ_x, σ_y, and τ_{xy} along the bonded area was determined as a function of the length of overlap to thickness ratio L/t.

It was found that for the isotropic adherends the stresses reached a maximum near the ends of the overlap and decreased towards the center. As L/t increased the distribution became more nonuniform. For the anisotropic adherends, in which the properties of the adherend was different for each orthogonal direction, the distribution did not show the oval two-peaked distribution but was instead essentially uniform. Both cases are compared in Figure 12.

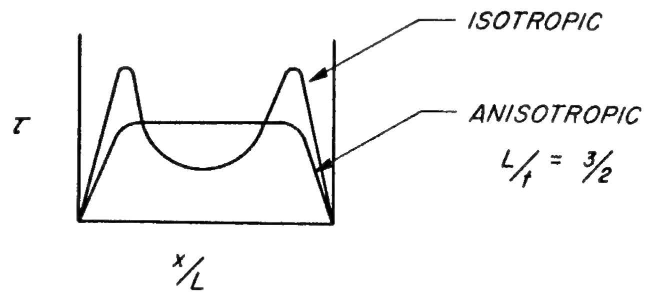


Figure 12.--Stress Distribution in a Double-Lap Joint with Isotropic or Anisotropic Adherends When Subjected to Compressive Loading.

The case of a single overlap joint was also investigated for isotropic and anisotropic adherends and the same general results were obtained. Again the anisotropic case showed a more uniform shear distribution than the isotropic.

Throughout this work the adhesive was assumed to be thin enough to be nonexistent; therefore, the analysis actually applies to the case of a monolithic specimen with an offset.

Misztal, I. (47)

Stress Distribution in Glued Single and Multijoints of Sheets Subjected to Shear Along the Splice Line. Bull. Acad. Polonaise Sci. Cl. IV, 4, pp. 21-27. 1956.

It was not possible to obtain this paper. It is included here for those who may have access to the reference. The following comments were made by Lubkin in Appl. Mech. Rev. 10, p. 293 No. 2086, 1957.

The article is an analysis of a long lap joint bonding together two thin sheets with a thin adhesive film. The joint was subjected to a uniform load per unit length parallel to the free edges of the adherends. The shear stress distribution was obtained for the adhesive layer and was found to reach a maximum at the ends of the joint. All materials were assumed to behave elastically.

Lubkin, J. L. (42)

The Stress Distribution in Adhesive Joints, Midwest Research Institute, Technical Report, Contract NOrd - 13383, 1954; also: J. Appl. Mech., Trans. Amer. Soc. Mech. Engrs. 24, p. 255, 1957.

This work was a continuation of an earlier study reported by Lubkin (41). The objective of this study was to present and analyze extensive calculations of the adhesive stresses in tubular lap joints (sleeve joints).

An analysis is presented for a tubular lap joint based on the assumption that the tubes can be treated as thin shells. The following assumptions were made.

(1) The adhesive has linear elastic behavior and acts as a layer of infinitesimal coil springs positioned between the shells. The adhesive was assumed to be sufficiently thin so that the stress distribution across the film thickness would be uniform.

(2) The cylindrical adherends can be treated by the standard theory of bending and stretching of thin shells.

(3) The applied forces are of a magnitude such that no finite deflections of the shell walls will occur.

It was pointed out that the tubular lap joint cannot be realistically formulated as one related to the flat-plate type lap-joint analysis of Goland and Reissner. The tubular lap joint does not undergo the same type of applied loads since it is not subjected to the bending moment found in the flat-plate-type joint.

Using the analysis developed, the stress distribution was calculated for 48 different tubular lap-joint configurations. The following dimensionless parameters were chosen to describe the joints:

(1) Adherend Poisson's ratio = 0.3
(2) Adhesive Properties - $E_a/G_a = 8/3$
(3) Overlap parameter $L/t = 1, 2, 5, 10$
(4) Elastothickness parameter - $\dfrac{t_a E}{t_1 E_a} = 4, 20, 100$
(5) Tubularity parameter - $\dfrac{t_1}{2d_a} = 0.01, 0.025, 0.05, 0.1$

The first four parameters are common to flat-plate lap joints but the tubularity factor is unique to tubular joints. The stress distributions were given for the normal stresses. σ_x, σ_y and the shear stress $_{xy}$ expressed as stress concentration factors $T = \sigma_{xy}/\sigma_a$ and $N = \sigma_x/T_a$. These calculations showed four important trends.

(1) N and T increased and their distribution along the overlap became more nonuniform as the length of overlap increased.

(2) The maximum N and T occurred at the loaded end of the inner adherend tube.

(3) T and M became more symmetrical about the center of the lap as the relative tube thickness $\frac{t_1}{2d_a}$ became smaller.

(4) As the adhesive became more flexible the stress distribution became more uniform and the stress level decreased.

Further calculations were made to determine the distribution of the principal stresses in the adhesive, and the maximum shear stresses. The same general trends were noted as stated before. A section was included on the design of tubular joints based on the assumption that the stress-strain behavior of the adhesive was known and that it was linear elastic.

The importance of determining the stress-strain behavior of adhesive materials was emphasized and suggestions made for a torsion-type specimen and a short overlap plate-type lap specimen to determine these properties.

This report is highly recommended as a reference for tubular lap joints and a general approach to the mechanics of adhesive joints.

Sherrer, R. E. (59)

Stresses in a Lap Joint With Elastic Adhesive. USDA Forest Products Laboratory Report No. 1864, 1957.

A theoretical analysis is presented for the stresses and displacements in a conventional lap joint bonded with an isotropic elastic adhesive and subjected to a tensile load. The adherends were assumed to behave as plates in bending which implies plane stress and the adhesive was assumed to be in a state of plane strain. A solution to the problem was stated in the form of an infinite series with which difficulties were encountered in obtaining convergence.

Stress distributions were obtained, but only that for stress normal to the plane of the joint was believed to be meaningful.

The basic model chosen appeared to require reformulation.

Ito, K. (33).

Note on Adhesive Strength Test by Lap Joint. Institute of Physical and Chemical Research (Tokyo) Scientific Paper 54, pp. 295-306, 1960.

An analysis is presented of a conventional lap-type joint. Elastic behavior was assumed and the overlap bond area was considered to behave as a solid beam, neglecting the presence of an adhesive film. The deflection of the

neutral axis of this solid beam area was determined for four different boundary conditions (load conditions). The maximum stress concentration and maximum deflection which occur at the ends of the overlap were determined for each load condition. These factors were strongly affected by the method of loading and illustrate the importance of applying the tensile load to a lap joint in the same manner when conducting strength tests.

The load conditions investigated are illustrated in Figure 13.

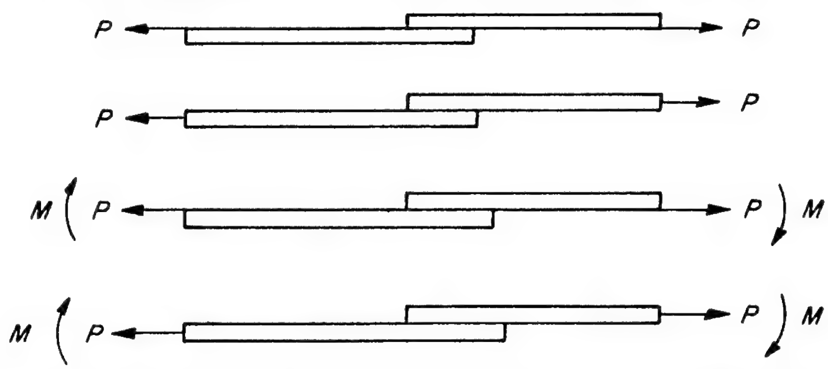


Figure 13.--Four Types of Loading Conditions Investigated in an Analysis of a Conventional Lap-Type Joint.

Albert, W. E. (1)

Stress Analysis of Bonded Joints. Martin Co., Orlando, Stress Analysis Memo No. 2.2, 1961.

An analysis is made of the shear strain distribution in the adhesive in a conventional lap joint assuming no bending of the joint. Loading of the joint was due to a tensile stress on the adherends or due to a linear temperature distribution along the joint. The analysis was the same as that presented by Arnovlevic (7). The usual shear strain distribution was obtained with the maximum strains occurring at the ends of the overlap.

Numerical computations were made for an actual joint consisting of 0.060-inch aluminum ($E = 10^7$ lbs./in.^{2}) bonded with a length of overlap of 1.00 inch with a 0.01-inch thick adhesive film of $G_a = 0.25 \times 10^6$ lbs./in.^{2}. For this condition the maximum stress concentration was 3.87.

In a later private communication with Albert it was pointed out that this analysis had been extended to a lap joint configuration in which the adherends were tapered in thickness throughout the bonded area. It was determined that the tapering had very little effect on the stress distribution, except when the thickness could be tapered down to zero thickness. In this instance a uniform shear distribution was obtained.

This point is important, since in actual bonding practice it is very difficult to obtain a taper to zero thickness.

Hahn, K. F., and Fouser, D. F. (30)

Methods of Determining Stress Distribution in Adherends and Adhesives. J. Appl. Poly. Sci. 6, pp. 145-149, 1962.

This paper was given at the "Symposium on Structural Adhesives" conducted at Picatinny Arsenal in September 1961. An analysis was made of the bending stress distribution in the adherends of a lap joint using the assumption that the adherends act as cylindrically bent plates. The moment causing the bending arises from eccentric application of the tensile load to the joint due to the offset in the line of force. The bending stress was superimposed on the tensile stress in the adherend to yield the maximum stress at any point in the adherend. This portion of the analysis required no knowledge of the adhesive film.

The second portion of the paper covered the analysis of a double lap joint in which the bending moment was neglected and only differential straining was considered. This analysis followed that of Arnovlevic (7) and the shear stress distribution in the adhesive was determined.

This paper summarizes the work covered in the earlier reports by Hahn (27, 28).

Lobbett, J. W., and Robb, E. A. (39)

Thermo-Mechanical Analysis of Structural Joint Study. WADD TR-61-151, 1962.

The purpose of this study was to investigate new thermo-mechanical analytical methods for high speed airframe bolted and brazed joint designs. The work concerned with brazed joints is of particular interest to the present review. An analysis was made of a continuous lap type joint by the Minimum Complementary Energy Method which is based on minimum energy principles of theory of elasticity and the Redundant Force Method which is based on a matrix method of structural analysis for statically indeterminate structures.

The lap joint model chosen for analysis by the Minimum Energy Method consisted of a monolithic joint in which the adherends and adhesive both have the same properties. The joint was considered to be loaded without bending. When this analysis was compared to results from a photothermoelastic analysis of a monolithic joint model, the stresses predicted were lower by 23 percent than those obtained experimentally.

The real contribution of this study is the application of a matrix structural analysis method to a lap joint idealization. With this advance, it becomes possible to use computer methods for calculating stress distributions in joints and to study joint discontinuities and geometry changes on a much finer scale. The method is based on the analysis of a joint model consisting of a network of bar elements and panel elements. The bars carry axial loads and the panels carry shear. The spacing of the elements can be varied according to the accuracy of analysis desired and their mechanical properties can be varied to conform to the proper materials behavior. With this technique any combination of adherend and adhesive mechanical properties can be chosen, the joint geometry can be varied, the adherends tapered or the shape of the extruded band line varied. The idealized discrete structure can be subjected to any desired distribution of internal or external load. It was possible to study geometrical details as fine as the shape of air-adhesive interface at the end of the length of overlap.

In order to test the validity of the Redundant Force Method an analysis was made of the joint previously investigated by Cornell (15). The analysis gave excellent agreement with the photoelastic results obtained by Cornell. The joint analyzed consisted of a 0.072-inch thick sheet bonded with a 0.005-inch braze layer to a thicker 0.102-inch sheet. A sketch of the joint idealization is shown in Figure 14.

Although only the elastic condition was treated, it was pointed out that the matrix structural analysis technique could be extended to materials with non-linear behavior.

Anonymous (6)

Design Study to Improve the Structural Efficiency of Polaris Filament-Wound Motor Cases. Goodyear Aircraft Corp. GER 10741, Navy Bureau of Weapons Contract NOw 61-0500-c (FBM) 1962.

This report describes the work carried out to improve the structural strength of a composite cylinder consisting of an interior cylindrical shell of thin aluminum bonded to an exterior shell of filament wound fiberglass. Shear failures were experienced between the two shells during internal pressureization of the cylinder when the structure consisted of simply winding the fiberglass on the aluminum without any special adhesive layer. In this instance the resin used to bind the fiberglass also acted as the bonding agent to the aluminum. The bond was failing due to differential straining between the fiberglass and aluminum.

An analysis was made of the stress distribution in the adhesive assuming a simple lap joint without bending. Using the analysis, an adhesive shear modulus and film thickness were chosen that could absorb the maximum shear strain expected.

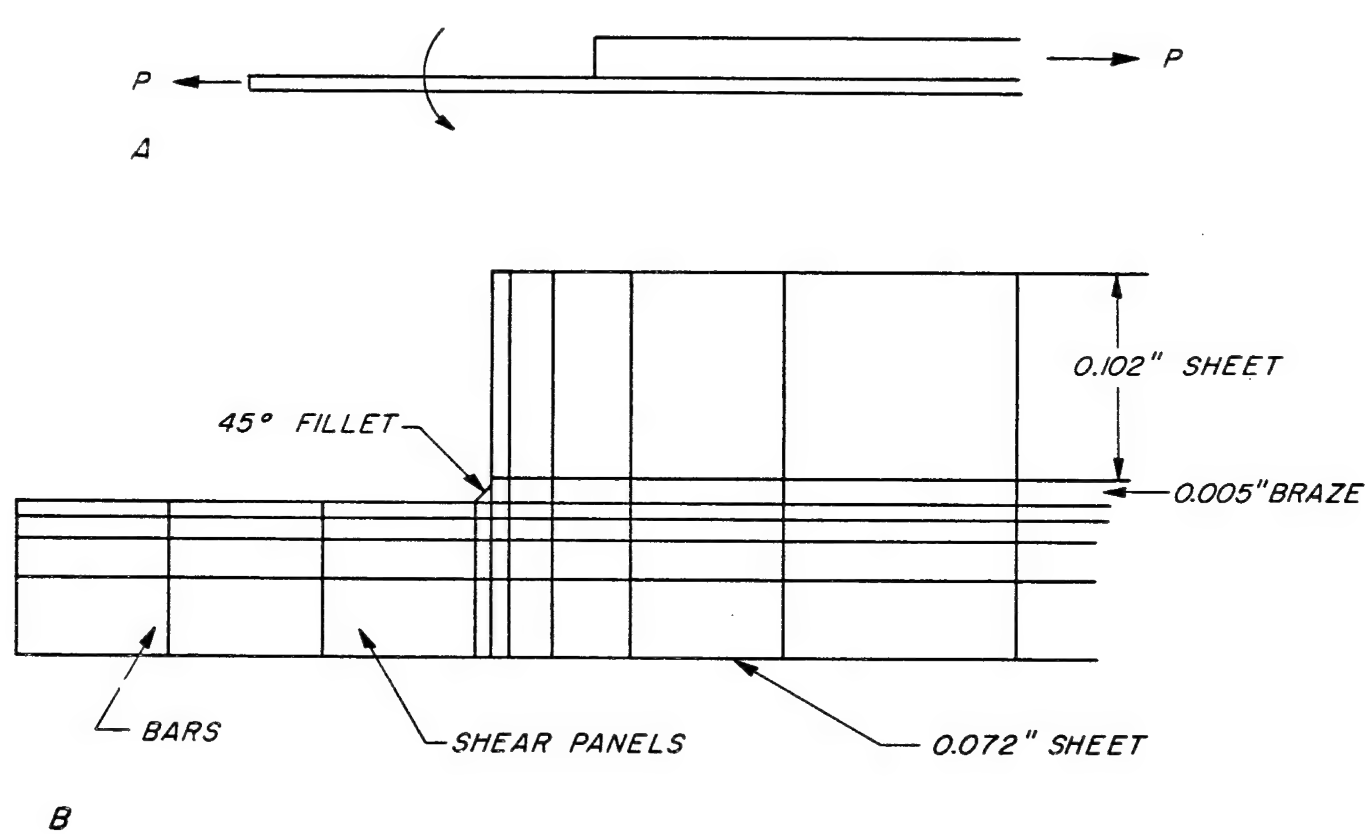


Figure 14.--A Continuous Lap-Type Joint: A, Actual Form; B, Idealization of Joint for Matrix Analysis.

It was then necessary to evaluate the shear modulus of the adhesives available. The shear modulus was determined using a double lap joint configuration with thick adherends. The shear modului of the adhesives in question were very low (100-300 psi); therefore, the thick adherends behaved as rigid bodies and it was assumed that a uniform shear distribution existed in the adhesive. By simply plotting adherend displacement as a function of applied stress, the shear modulus could be determined with sufficient accuracy. In no instance was a linear stress strain curve obtained therfore the secant modulus was used.

Cylinders fabricated with the interlaminar elastomeric adhesive behaved satisfactorily. This report covers one of the few instances where any stress analysis work has been applied to an actual adhesive joint design problem.

Goodwin, J. F. (26)

> Research and Thermomechanical Analysis of Brazed or Bonded Structural Joints. Progress Reports 1-10, May 1962 -February 1963 on Air Force Contract AF 33(657) - 8542, conducted by Douglas Aircraft, Santa Monica, Calif., also ASD-TDR-63-447, same title, September 1963.

This work is the most recent and up to date study concerned with the analysis of the lap-type joint. The primary purpose was to analyze brazed joints, but the work is directly applicable to adhesive bonded joints. The major difference was that in a brazed joint the mechanical properties of the braze may be equal to the sheet properties or may only be smaller by a factor of two or three.

Goodwin has used a matrix structural analysis technique for indeterminate structures and applied it to a model of a joint. The model is a two dimensional framework of bars and shear panels. By choosing a proper size or fineness of the grid, using mechanical properties of the bars and shear panels to coincide with adherends and adhesive in the respective areas, it was possible to completely idealize a joint and subject it to any desired loading situation.

By using this matrix formulation of the problem, all the simulataneous equations involved in this method are placed in a language ideal for the digital computer. The computer then allows one to make many more extensive calculations than could even be considered using other analysis methods.

The work was divided into three sections:

1. Investigation of various joint parameters on the elastic stress distribution in the joint.

2. Development of a theory of fracture for lap joints under fatigue loading.

3. Verification of the fracture theory through actual testing of specimens.

Each section will be reviewed separately.

1. Investigations of various joint parameters on the elastic stress distribution.

Using the matrix analysis the shear stress and normal stress distribution parallel to the sheet were determined. A typical joint consisted of 0.040-inch sheet material bonded to 0.80-inch base bar through a length of overlap of 0.70-inch with a 0.003-inch braze layer. The ratio of sheet modulus E_1 to braze modulus E_a was two. An example of the joint idealization and complexity of the stress distribution is shown in Figures 15 and 16.

The effect of different joint parameters on the stress distribution were expressed as changes in the maximum shear and tensile stress concentration factors. The maximum stress concentrations were always found to occur at the ends of the length of overlap.

n_s = shear stress concentration in the braze at the end of the overlap
n_t = tensile stress (parallel to joint) concentration in the adherends at the end of the overlap.

The following are general interpretations of the data.

(a) Joint Bending - in a joint allowed to undergo bending (unsupported), n_s increased and n_t increased.

(b) Braze Modulus - when G_a decreased 55 percent, n_s and n_t decreased 3 to 14 percent.

(c) Adherend Taper - in a joint with one adherend tapered throughout the length of overlap, n_s decreased 34 percent and n_L decreased 13 percent.

(d) Braze Thickness - t_a increased 128 percent, n_t decreased 2.3 percent and n_s decreased 4.7 percent.

(e) Length of Overlap - L/t increased 17.5 to 50, n_s increased 180 percent, n_t decreased 1.8 percent.

(f) Braze fillet radius (air-braze interface) - increasing the radius of curvature 0.075t to 0.825t, n_s decreased 33 percent, n_t decreased 31 percent.

All the above were changes in elastic stress concentrations. Through proper formulation of the problem the analysis could be extended to cover plastic behavior as well.

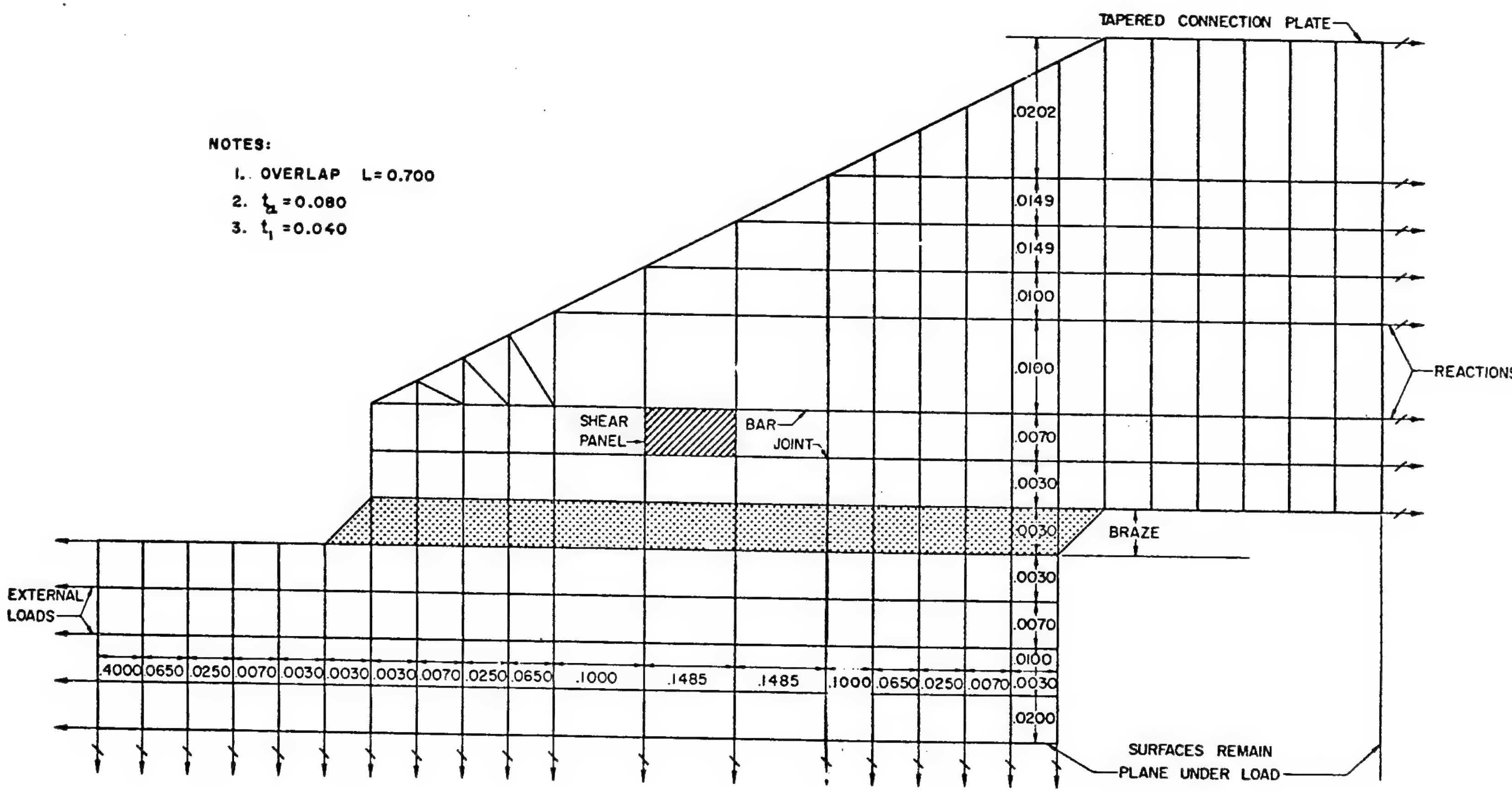


Figure 15.--Continuous Brazed Joint Idealization for Matrix Analysis.

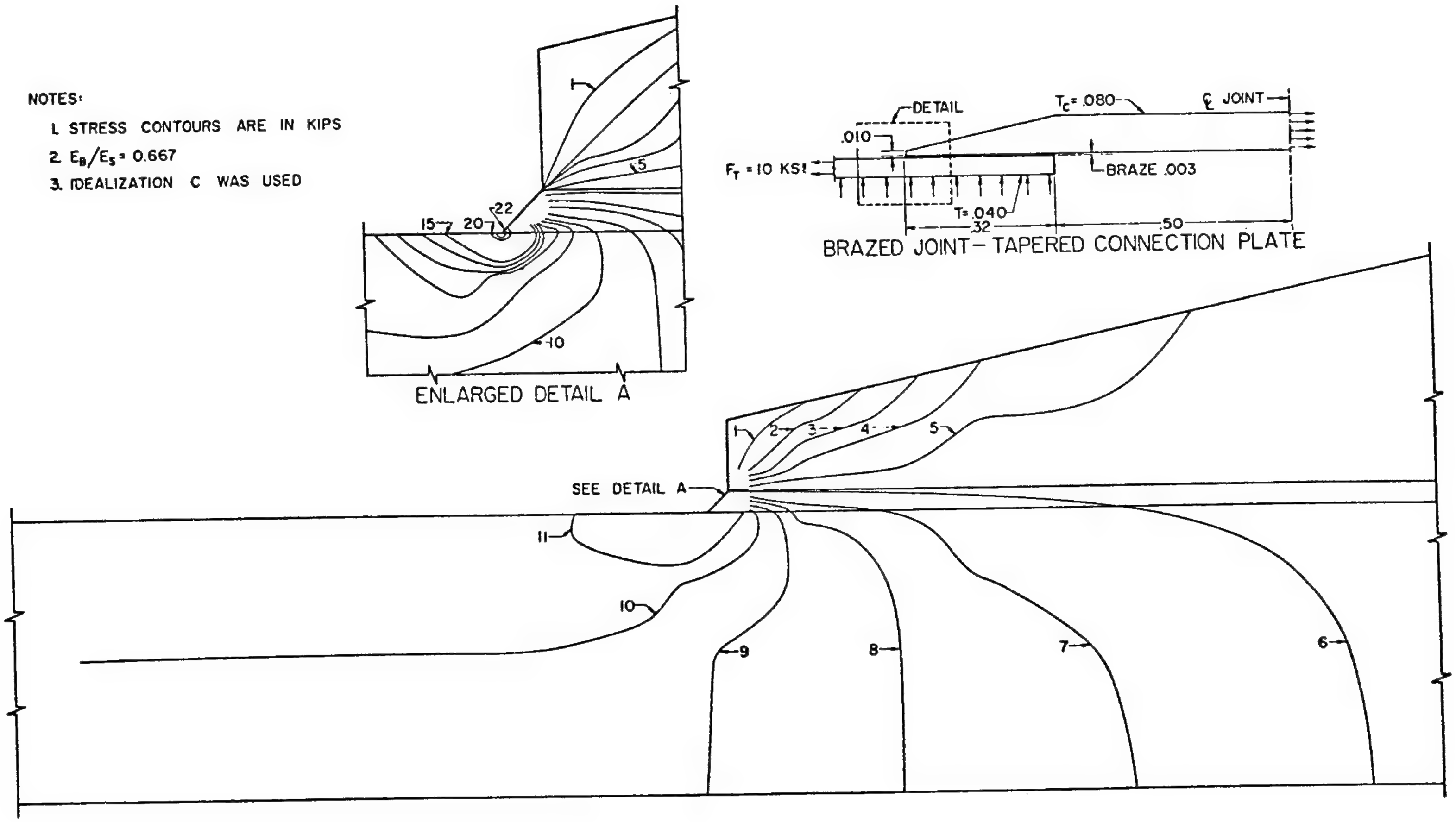


Figure 16.--Typical Brazed Joint Detail Showing Axial Tension Stresses in Sheet.

2. Theory of Fracture.

It was noted that in the static testing of several double lap-type joints that the joint strengths (average shear strength of braze) exceeded the expected shear strength of the braze alloy. It was postulated that the braze alloy must be exhibiting some degree of plasticity. To check this possibility the following reasoning was used:

Assume that when the braze in the joint reached its proportional limt at some point, the joint would fail.

At this point

$$\tau_{a_{PL}} = \tau_{mPL}/n_e \tag{54}$$

Using the matrix analysis a relationship was obtained for n_e the elastic stress concentration as a function of (L/t).

$$n_e = \frac{AL}{t} + B. \tag{55}$$

Substituting,

$$\tau_{a_{PL}} = \frac{\tau m_{PL}}{A\, L/_t + B} \tag{56}$$

When the curve of this function was compared with actual joint strength plotted as a function of L/t they both had the same general form. In this instance the τm_{PL}, the proportional limt strength of the braze, was determined on free films of the braze alloy. It appeared that the results of the elastic matrix analysis could be used to predict strength of joints, provided some information was available for the shear strength of the adhesive.

The following discussion pertains to double lap joints or joints in which only differential straining is occurring.

It was assumed that the braze in the joint was undergoing plastic flow prior to joint failure and that when this width of the plastic zone (W) reached some critical valve (W_c) the joint would fail. This width (W_c) was assumed to be a material property of the braze. This concept is illustrated in Figure 17.

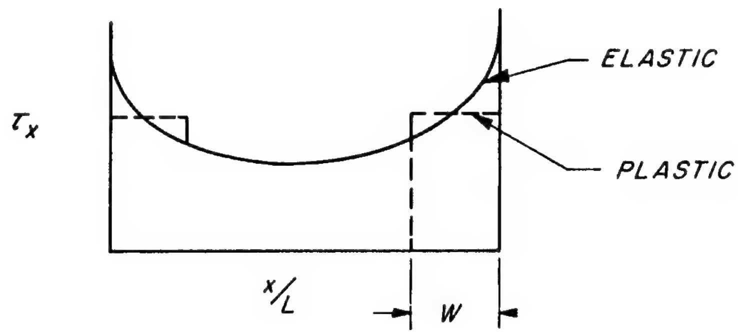


Figure 17.--Elastic and Plastic Stress Distribution in Double Lap Joints in Which Differential Straining is Occurring.

First the form of the elastic shear stress distribution was determined. This was obtained by using the Arnovlevic-Volkerson type analysis for joints with differential straining only.

$$\tau_x = \frac{P\lambda}{2} \frac{\cosh \lambda x}{\sinh \lambda L/2} \tag{57}$$

At the end of the overlap $x = L/2$ and $\tau_x = \tau_{max}$ we can obtain

$$n = \frac{T_{max}}{T_{and}} = \frac{\lambda L 2}{\tanh \lambda L/2} \tag{58}$$

The parameter λ is related to the adherend-adhesive thicknesses and the adherend-adhesive mechanical properties, and will determine the general shape of the stress distribution. Instead of using the joint properties to obtain λ, it was obtained by assuming n_e equal the shear stress concentration factor, obtained from the matrix structural of the same joint and then solving for λ. In this instance n_e represents the elastic stress concentration factor.

Now having the proper shape of the elastic shear distribution curve and knowing the proportional limit shear stress of the adhesive, the superimposed plastic distribution could be indicated. The width of the plastic zone (W) was obtained by equating the area under the plastic stress distribution to the area under the elastic stress distribution it had replaced. The following expression was then obtained for the strength of a joint (T_a):

$$\tau_a = \eta\, \tau_m \tag{59}$$

$$\eta = \frac{\beta / \alpha}{1 - \cosh\beta} + \sinh\beta/\tanh\alpha \tag{60}$$

$$\alpha = \lambda L/2 \tag{61}$$

$$\beta = \lambda W_c \tag{62}$$

λ was obtained using the matrix-structural analysis. τ_m and W_c are material parameters of the braze and must be obtained from test data W_c can be calculated using some actual joint strengths to obtain Ta/T_m, calculating λ for the joint from the matrix analysis and then, with a graph of η as a function of α for different valves of β, calculate W_c.

When this Constant Plastic Zone theory was used to predict joint strengths it was found to have excellent agreement with test data at temperatures of 75°, 650°, and 850° F.

Several other minor studies connected with this work yielded the following points of interest:

1. The question had been raised as to whether the braze in the joint has different properties from that in some free form. Some composite beam specimens were fabricated utilizing the braze alloy. They were tested in bending and their performance compared to that predicted by analysis. The results indicated that the braze did not change in modulus of elasticity when incorporated into the composite beam.

2. Elastic stress concentrations calculated from the matrix analysis were used to attempt to predict joint behavior under fatigue loading. The marked effect of fillet radius on elastic stress concentration appeared to have no effect on fatigue life of a joint. It was concluded that small imperfections of a size much smaller than the fillet radius were affecting fatigue behavior, and that plastic yielding at the ends of the overlap was probably reducing the effects of fillet radius.

This work covers the most extensive investigation of factors affecting stress distributions in bonded joints. The techniques developed should be applied to adhesive joints to reinvestigate these many factors, as they might affect the behavior of joints bonded with organic materials.

III. EXPERIMENTAL STRESS ANALYSIS OF BONDED JOINTS

The problem of experimentally determining the stress distribution in a bonded lap-type joint is not a simple one. Ideally, one would like to know the stress distribution in the adhesive film and the adherends. If one considers a conventional 0.500-inch lap joint of 0.032-inch thick aluminum alloy bonded with a conventional adhesive of film thickness 0.010-inch, the size of the specimen is such that very little adhesive material is available for experimental manipulation.

Consider the problem of determining the stress distribution in the adhesive. The only exposed adhesive is a 0.010-inch line at the edges of the joint. This represents only a minute fraction of the total adhesive within the joint, the rest being obscured by the opaque adherends. It would be difficult to apply any current type of external extensometer or strain gauge to this thin 0.010-inch line. The question can also be raised as to how meaningful this information would be if it were experimentally possible. A solution might be to use the adhesive itself as the strain gauge and in some manner penetrate the opaque adherends, or use some technique to penetrate through the adhesive film in the direction parallel to the plane of the joint.

Because of the experimental difficulties involved, most studies have been conducted on large models of a joint; then using the principle of similarity, assume that the results obtained in the model will also be true in an actual joint. In order to properly simulate an adhesive joint, the following conditions should be satisfied by the model:

(1) Simulate the relative ratio of adherend thickness to adhesive thickness.

(2) Simulate the relative ratio of adherend shear modulus to adhesive shear modulus or the general ratio of mechanical properties of adherend to adhesive.

(3) Be large enough in the direction simulating the joint width to assure a condition of plane strain in the plane of the adhesive.

(4) Simulate a true adhesive bond between the adherend and adhesive. It is important to simulate the discontinuous nature of the mechanical properties at the adherend-adhesive interface.

There is one final point that should be considered when analyzing the results of model studies, namely: If one simulates an adhesive joint by geometrically enlarging the joint to a large model, can the enlarged adhesive, in particular, be expected to behave as it would in the form of a very thin film? Is it a valid assumption that an adhesive 0.005 inch thick behaves the same mechanically as it does in some bulk form 0.500 inch thick. No references were found concerning this point though it appears repeatedly in discussions.

The earliest work related to lap-joint analysis is that conducted by Andrade (2) on a large rectangular block of gelatin subject to pure shear. The gelatin block was 4 by 4 by 16 inches in size and was bonded to two wooden boards along two of the opposite long sides. A shear load was applied to the block through the wooden "adherends." Shear strains were determined by measuring change in a gridwork of reference marks placed on the gelatin surface. Although the purpose of this study was not related to the analysis of joints, the geometry and experimental condition of the study make it analogous to a simple lap joint without bending. The study satisfies all the criteria for an

acceptable experimental analysis except for size. The ratio of the modulus of the wood to the modulus of gelatin was approximately 10^5, which is close to that of a conventional metal joint bonded with a flexible organic adhesive.

Andrade found the shear strain to be a maximum near the edge of the overlap and a minimum towards the center. Of particular interest was the fact that there was a variation in shear strain across the adhesive from one adherend to the other. An inspection of the shear distribution curves showed the maximum shear strain concentration to be approximately 1.25.

It was of further interest that Andrade obtained any nonuniform distribution at all in the gelatin. The modulus of wood was very high compared to the gelatin so the wood must have acted almost like a rigid body. The shear stress was very low (0.002 pounds per square inch) and the shear loading was applied in a manner to prevent the development of any peel forces; also, the gelatin was very thick. One would intuitively expect all these factors to lead to a very uniform shear distribution.

The earliest photoelastic work conducted relative to joints was that of Coker (14). Again, the purpose of the work was not to study lap joints, but the physical arrangement of the experiment is of special interest. Coker used a single sheet of plastic to which he bolted three parallel sets of steel bars, so that the model actually simulated a double lap joint (Fig. 18). Using transmission photoelasticity, the shear strain was determined as a function of film thickness and length of overlap. Again it was found that the shear strain increased from zero at the end of the overlap to a maximum near the end, and then fell to a minimum toward the center. These results were obtained from measurements along the centerline of a bond area and this result appeared to be unaffected by changes in adhesive film thickness. The film thickness did appear to have an effect on the distribution near the ends of the overlap. In this study, the ratio of (E) of the steel to that of the plastic was 10.

Tylecote (63) continued the photoelastic studies by investigating the stress distribution in plastic models of spot-welded joints. The models were cut from single plane sheets of plastic. The homogeneous joints were comparable to an adhesive-bonded joint in which adhesive and adherend both had the same mechanical properties. Tylecote found both high shear and normal forces near the ends of the overlap with a maximum shear-stress concentration factor of 5.7. These studies show that the offset in the geometry of the lap joint had a strong effect on the stress concentration in the joint, since in these studies there was no discontinuity in mechanical properties within the joint.

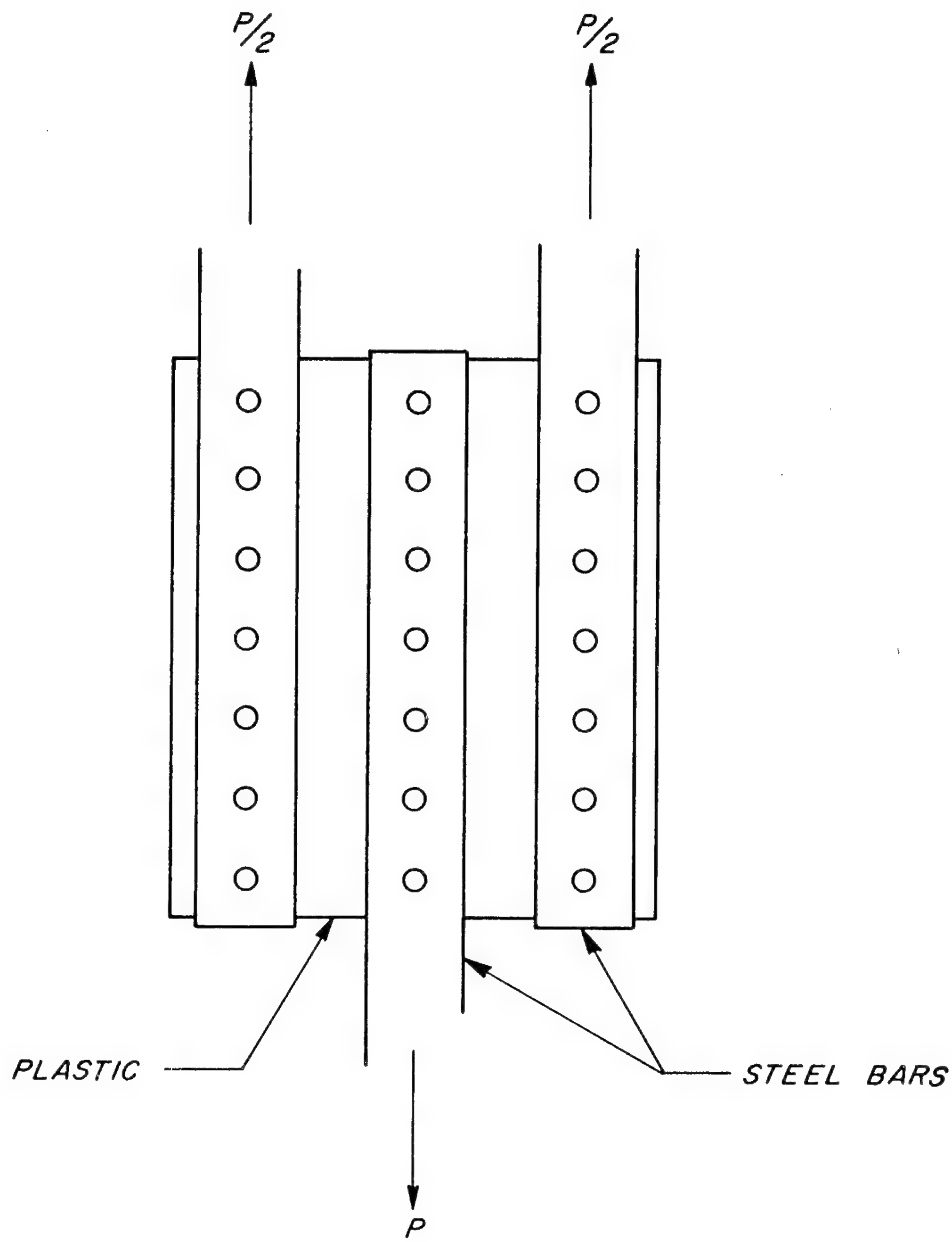


Figure 18.--Photoelastic Model Simulating a Double Lap Joint Used by Coker (4).

Another experimental approach to the problem was used by Jackson (34). This involved making models of lap joints cut from pieces of low-modulus rubber. Jackson used this technique to graphically illustrate in a qualitative manner how the strains in lap joints were affected by changes in joint geometry. In this instance the joints were homogeneous, of a single material and the strains illustrated were actually those occurring in the adherends.

In 1948, Mylonas (48) published the first of his classic photoelastic studies of the stress distribution in composite models. Initially Mylonas attempted to make some joint models using wood adherends bonded with a thick layer of Catalin 800 plastic. He had difficulty making satisfactory joints that were stress free because of the high shrinkage and a skin effect during cure of the resin. No extensive analyses were made of the wood-plastic joints. Mylonas then made some models in which the joint was cut from a single sheet of Catalin 800, and steel strips were bolted to the plastic in the adherend areas to stiffen the plastic and better simulate a joint (Fig. 18). In these models the adhesive-air interface was made circular and the major conclusion of the study was that this interface, at the ends of the length of overlap, was the most critically stressed area and required further study. Maximum stress concentration factors were calculated for the models using both the Volkersen (64) and Goland and Reissner (24) analyses and these were found to be generally lower than those observed photoelastically.

Norris and Ringelstetter (52) made some shear-strain measurements along a glueline in a model simulating the bond between the skin and cap strip in a wooden aircraft wing. The model geometry was that of a double lap joint and shear strain measurements were made with a single Tuckerman optical strain gage placed successively at different positions along the bond line. The measurements indicated a maximum shear strain at the ends of the overlap.

Mylonas (49) in a continuation of his first study, conducted further photoelastic work on the stress distribution along the air-adhesive interface. In this instance the models were made by casting a mixture of ethoxylene resin, cyclohexanol and dibutylpthalate between 1/2-inch-square bars of Bakelite reinforced with steel. The actual adhesive was simulated by a cast block of resin 1/2 by 1/2 inch in cross section. It was found that as long as L/t was longer than 3, the length of overlap of the joint had no effect on the stress distribution at the ends of the overlap. With this restriction joint models were fabricated in which the air-adhesive interface had different radii of curvature and different slopes with respect to the line at the joint.

It was concluded that for joints with curved air-adhesive interfaces at the ends of the joint, an interface with a small radius would most likely fail cohesively in the adhesive, and for larger radii the joint would fail along the adherend-adhesive interface. For a straight air-adhesive interface the stress was

highest for an angle of 90°, of adhesive to adherend and this decreased as the angle decreased. Refer to Figure 22. This study emphasizes the importance of the joint geometry at the ends of the overlap and could serve as a sound basis for further work in this area. The only question that can be raised concerning the work is how well this relatively thick bond line simulates the behavior of a much thinner adhesive film. Mylonas pointed out that a complex three-dimensional stress distribution was probably present in the adhesive.

In another study using the rubber-analog technique, Demarkles (18) investigated the effect of applied load on the stress distribution in a lap-type joint. The models were made by bonding together two pieces of foam rubber with an adhesive of the same mechanical properties as the rubber. Strain measurements were made by measuring the deformation of a grid painted on the surface of the rubber. The measurements were compared with results predicted by a Volkersen (64) type analysis. The stress distribution determined was that present in the adherend rather than the adhesive. The shear stresses predicted by the analysis were higher than those determined experimentally.

The most complete study of the effect of bending on the stress distribution was made by McLaren and MacInnes (46). The study was based on the photoelastic analysis of composite models of lap joints made from two different birefringent plastics and also of single-sheet homogeneous models. In the composite models the plastic used to simulate the adherends was 20 times stiffer than that used for the adhesive. A bending moment was simulated in the joint area by applying a load to the model at varying angles to the length of the joint. Refer to Figure 23. With this technique it was possible to apply a moment to the bonded area similar to that occuring in an actual joint and then also to reverse this moment to compensate for the moment caused by joint geometry.

The study indicated in all instances that the ends of the overlap showed the maximum isochromatic fringes, and that these decreased in order toward the center of the overlap. Also, as the positive bending increased, the shear stress increased; but in the case of negative bending it was actually possible to decrease the maximum shear stress at the ends of the overlap. Based on these results, a specimen configuration was suggested that would eliminate any bending and simply subject the lap joint to pure shear stress.

The only work uncovered which dealt directly with the stress distribution on actual joints was the work of Hahn (27). Hahn used a reflective photoelastic technique to study the stress distribution as a function of applied load on some aluminum lap joints bonded with Metlbond 4021. The adherends were 1/4 by 2 by 8 inches in size and the length of overlap was 2 inches. The bond area

was 2 by 2 inches square. Hahn was interested in experimentally determining the stress distribution in the adherends just adjacent to the bond area. The experimental stresses obtained compared very well with a mathematical analysis developed on the assumption that the adherends behaved as thin plates bent to a cylindrical surface. The most interesting part of the experimental work was that it showed the adherends to assume an antielastic shape just outside the bond area and this induced shear stress concentrations at the edges of the adherend. This means that the shear stress distribution is not uniform across the width of the adherends. The experimental work further showed that this antielastic curvature was also present within the bonded area. This would indicate that the stress distribution in the adhesive not only varies from one end of the length of overlap to the other, but also across the width of the bonded area, especially at the ends of the overlap. From the practical standpoint this means that a joint 2 inches wide is not twice as strong as a joint 1 inch wide. This was the first experimental indication of this type of behavior in joints and should be further investigated.

In a preliminary study, Kutscha (37) adopted the technique used by Mylonas (48) and made some photoelastic investigations of shear stresses in composite models of lap joints. The purpose of the study was to attempt to more closely approximate an actual joint, especially in respect to film thickness. Lap joints of 0.064-inch-thick aluminum alloy bonded with liquid Photostress A, a commercial photoelastic plastic, were made using a film thickness of 0.029 inch. The joint width was 0.25 inch. The shear stress distribution was determined in the adhesive film as a function of length of overlap and then compared to the distribution predicted by the Goland and Reissner (24) analysis. The experimental stress concentrations were found to be generally higher than those obtained analytically. There is some question concerning this comparison, since the shear modulus used for the adhesive in the analysis was determined on bulk material rather than on a film in a joint.

To summarize the experimental studies, it can be stated that the occurrence of elastic stress concentrations at the ends of the overlap have been qualitatively demonstrated, and in some instances somewhat quantitatively related to the mechanical properties of the adhesive. A nonuniform-stress distribution across the width of the joint has also been demonstrated. The important factors yet to be shown are: The stress distribution in the joint beyond the elastic limit of the adhesive and adherends, the effect of decreasing film thickness on the stress distribution in the adhesive, and the relationship between stress distribution throughout the joint and the ratio of mechanical properties of the adhesive to the adherend. The exact mode of fracture of adhesive joints has yet to be shown experimentally. The complexity of the stress distribution is shown by the qualitative sketch in Figure 19. The shear stress in the xz plane and the normal stress in the xy plane are indicated.

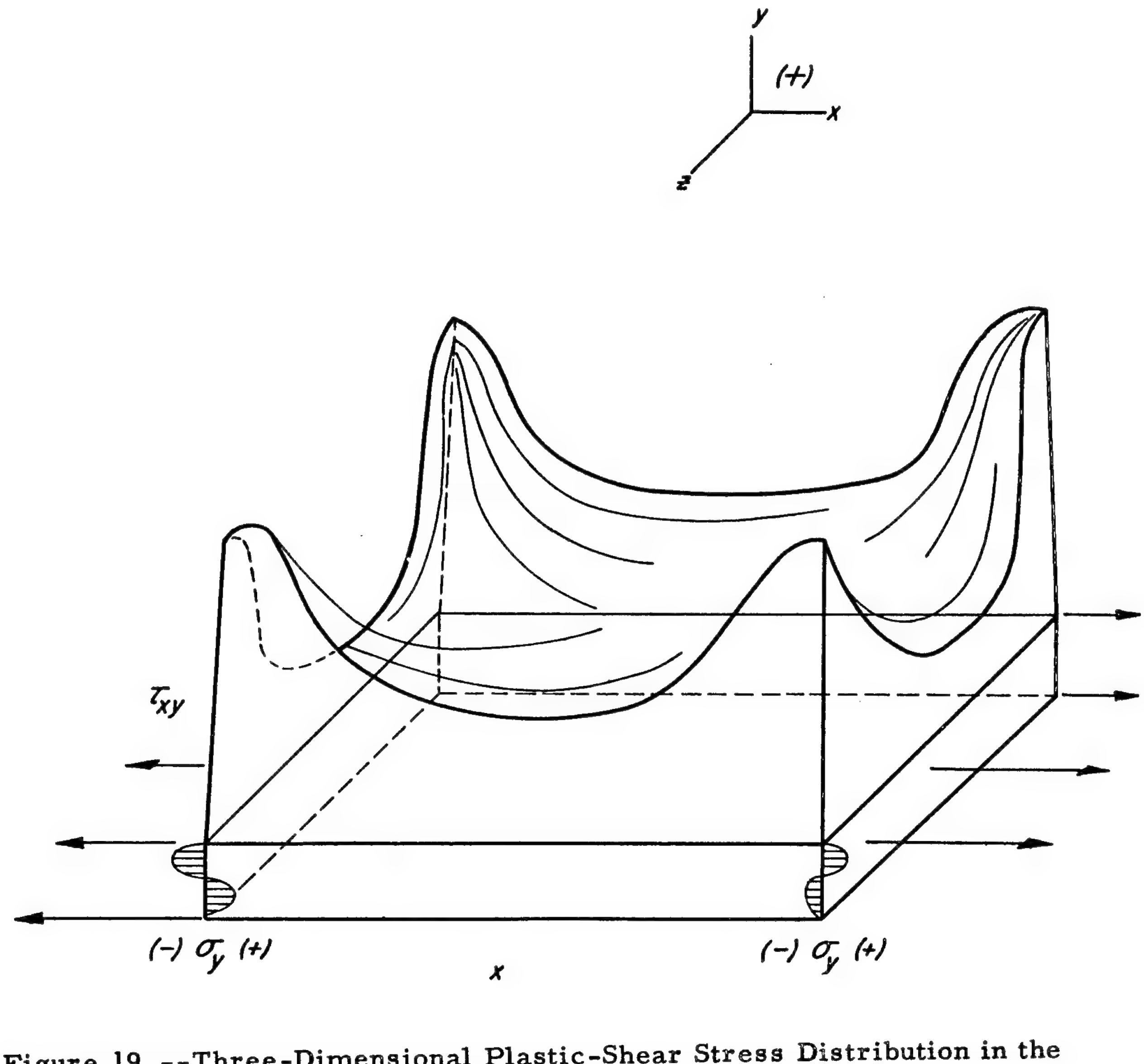


Figure 19.--Three-Dimensional Plastic-Shear Stress Distribution in the Adhesive Film of a Lap Joint.

A. Annotated Bibliography

Andrade, E. N. da C. (2)

The Distribution of Slide in a Right Six-Face Subject to Pure Shear. Proc. Royal Soc. (London) A85, pp. 448-461, 1911.

This work presents an experimental and analytical investigation of the shear strain distribution in a rectangular block of gelatin subject to pure shear load. The gelatin block was 4 by 4 by 16 inches and was sheared in the direction of the long axis by two wood boards fastened to the bottom and top of the gelatin. The block was loaded in such a manner to prevent any normal forces between the boards. The block was loaded only within the elastic limit of the gelatin. The average shear stress used was 0.062 psi.

This study was made to investigate the shear stresses affecting the stability of dams, but the experimental model is also analogous to the adhesive joint problem in that the adherends are infinitely stiff compared to the adhesive and the joint is loaded in shear only, with no peeling or normal forces.

The shear strain distribution was determined by inserting needles arranged on a 1 cm. square grid into one side of the block. Utilizing any three adjacent needles, a microgoniometer was used to determine angular deflection in the block under load. The shear strain was plotted as a function of position along the block (position in the overlap) at a given distance from the bottom adherend (position across the adhesive film).

The shear strain was found to be minimum at the center of the block, rise to a maximum near the ends, and then fall to zero at the edges. The shape of this distribution curve varied with the distance from the adherends, the maximum peaks being the most pronounced along the centerline of the block and the distribution becoming more uniform closer to the adherends. Even though the distribution was the most irregular along the centerline, the maximum values of shear strain were not observed at this line. Also, there were indications of secondary maxima occurring along the curves at positions away from the block ends.

The analysis of the system was formulated as a problem in plane strain, according to Love (40) in his "Mathematical Theory of Elasticity" Chapter IX. The general shape of the distribution curve, found experimentally, was reproduced analytically but the agreement was not very good.

The importance of this study is that it showed the shear strain distribution to be different from the parabolic distribution usually assumed in engineering practice at that time. Also it is important in showing that shear strain varies

through the thickness of the block from one adherend to the other, even for this very simple loading condition. It is also of interest that a nonuniform distribution was observed for this joint, in which the ratio of modulus of elasticity of adherend to adhesive was approximately 10^5.

$$E_{wood} \Big/ E_{gelatin} = \frac{15 \times 10^5}{15} = 10^5.$$

Coker, E. G. (14)

An Optical Determination of the Variation of Stress in a Thin Rectangular Plate Subjected to Shear. Proceedings Royal Society (London), 86(A587):291-319, 1912.

A photoelastic analysis was made of the stress distribution in thin sheets of cellulose-nitrate plastic under a shear load, as shown in Figure 20.

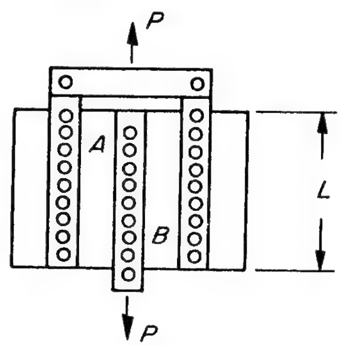


Figure 20.--Specimen Used for Photoelastic Analysis of Stress Distribution in a Thin Cellulose-Nitrate Plastic Sheet Subjected to Shear.

Three sets of steel bars were clamped to the sheet as shown with bolts causing a shear load on the unclamped areas A and B. Note that this arrangement is analogous to a double lap joint. The areas A and B represent the adhesive film thickness and the steel bars the adherends.

The stress distribution was studied in the areas A and B as a function of the length of overlap (varied from 2 to 10 inches), the adhesive thickness or distance between steel bows (varied from 0.5 to 2 inches), and applied load. The stress distribution was measured along the centerline between the steel bars.

One of the purposes of this study was to investigate the validity of the assumption that a parabolic shear distribution exists in structures of this type. It was determined that a parabolic distribution is obtained only for a few combinations of length of overlap and film thickness.

In general the shear stress rises to a maximum very rapidly from zero at each end of the plate and then decreases to a minimum towards the center. The maximum stress occurs at a distance equal to less than the film thickness of the adhesive from the end of the overlap.

The ratio of the mechanical properties of the adherend to adhesive in this instance was:

$$E_{steel} / E_{plastic} = \frac{3 \times 10^6}{3 \times 10^5} = 10.$$

Coker stated that for a relatively long rectangular plate the shear stress distribution was approximately uniform for the central section and then fell rapidly to zero at the ends. This general relationship was unaffected by changes in adheisve-film thickness, but the shape of the shear distribution at the ends of the overlap varied with film thickness.

The importance of this paper is its analogy to an adhesive joint. It represents the first attempt at an analysis in which the ratio of mechanical properties of adherend to adhesive are in the proper range, although the adhesive film thickness is rather large compared to the adherend thickness.

Tylecote, R. F. (63)

Spot Welding. Part II. A Photoelastic Investigation on the Stress Distribution in Spot Welds. Welding Journal 20:359s-368s, 1941.

A photoelastic stress analysis was made of models of single spot-welded joints. The models were cut from 0.12-inch-thick sheets of Xylonite (cellulose nitrate) and were four times larger than an actual joint. The models corresponded to a section parallel to the length of the joint and passing directly through the center of the spot weld. This would be analogous to an adhesive-bonded lap joint in which the adhesive and adherend had the same mechanical properties. The shear stress and normal stress distributions were determined from the isocinics and isochromatics.

The analysis indicated that the highest stresses occurred at the edges of the weld with stress concentration factors of 5.7 for shear stress and 4.5 for the normal (peel) stress perpendicular to the plane of the joint. There is some question as to how well an isotropic model of the type used simulates a welded joint. The Brinell Hardness was checked throughout some weldments which had been sectioned and a wide variation was found in this property throughout the joint. In some cases the adherend adjacent to the weld was softer than the

weld itself. Variations such as this would have a marked effect on the actual stress distribution in the joint.

The interesting point of studies such as this, concerning continuous-homogeneous joints, is that even when the adhesive and adherend are exactly matched in mechanical properties one obtains a non-uniform stress distribution throughout the bonded area; therefore, matched adhesive-adherend properties are not a solution to the problem of stress concentrations in lap joints, as at times has been suggested.

Jackson, C. C. (34)

Joints - Lap, Efficiency, Graphical Study of Physical Factors. Chrysler Corporation Engineering Report No. 4904, Feb. 1943.

A study was made to determine the effect of various physical parameters on the efficiency of lap joints through the use of rubber models, on which the strains would appear on a magnified scale.

Low modulus rubber was used in making models of a simple lap joint, a double lap joint, a double lap with the outside adherends tapered, and a scarf joint. In these models the adherends had the same properties. In addition two scarf joints were made in which the adherends were not the same.

Reference grids were placed on the models and photographs taken before and after loading to illustrate the deformations.

This method gives a good indication of what occurs in the adherends of the joints, but not in the adhesive, since the adhesives have the same properties as the adherends. No conclusions were given.

Mylonas, C. (48)

On the Stress Distribution in Glued Joints. Proceedings 8th International Congress of Applied Mechanics, London, pp. 137-149, 1948.

Work was carried out to determine the stress distribution on a composite model of a glued joint using photoelasticity, in which the adhesive itself acted as the photoelastic material. Photoelastic tests were first made on some joints in which Catalin 800 was used to bond together some wood adherends. The joints were 2 inches long with a film thickness of 1 inch. Difficulties were encountered due to the high residual stresses resulting from the shrinkage of the resin and it was not possible to fully analyze the joints. A pronounced skin effect due to irregular setting of the adhesive was also encountered on the surface of the adhesive. In spite of these problems, an

analysis of the stresses along the air-adhesive interface indicated that the maximum stress occurred approximately one-fifth the distance across the adhesive film, and that it was a tensile stress. The stress varied along the interface from tension to compression on the interface.

Other studies were made of models of lap joints, that were cut from a single sheet of plastic and in which the adherend portions of the models were reinforced with steel strips bolted to the plastic (Fig. 21). The models had an adhesive film thickness of 1/2 inch and the adhesive-air interface was semicircular. The maximum tensile stress on the interface occurred at a point .14 t across the adhesive film.

The maximum stress concentrations found in the model were compared to those predicted by the Volkersen (64) analysis and the Goland and Reissner (24) analysis, and were found to be higher than those predicted.

It was concluded that although the models did not completely duplicate an actual joint they did show that the adhesive-air interface was the critical area, and that this needed more study.

Norris, C. B., and Ringelstetter, L. A. (52)

Shear Stress Distribution Along Glue Line Between Skin and Cap Strip of an Aircraft Wing. NACA TN 2152, July 1950.

Shear strain measurements were made along the glue line of a specimen that was essentially a double lap joint made of Sitka spruce. The length of overlap varied from 6 to 2 inches and the outer adherend thickness from 3/4 to 3/16 inch. Strain measurements were made using a Tuckerman optical gage with a gage length of 1/4 inch placed at 45° across the glue line. The strain was determined at 1/4-inch intervals along the length of the specimen.

The measurements indicated that the strains increased rapidly from zero to a maximum at the re-entrant corner, gradually decreased throughout the overlap, and than rapidly fell to zero at the other end of the joint.

Mylonas, C. (49)

Experiments on Composite Models with Applications to Cemented Joints Proceedings Society Experimental Stress Analysis, 12:129-142, 1954.

This paper was a continuation of the earlier studies by Mylonas (48) and covers further attempts to make stress-free composite models of lap joints.

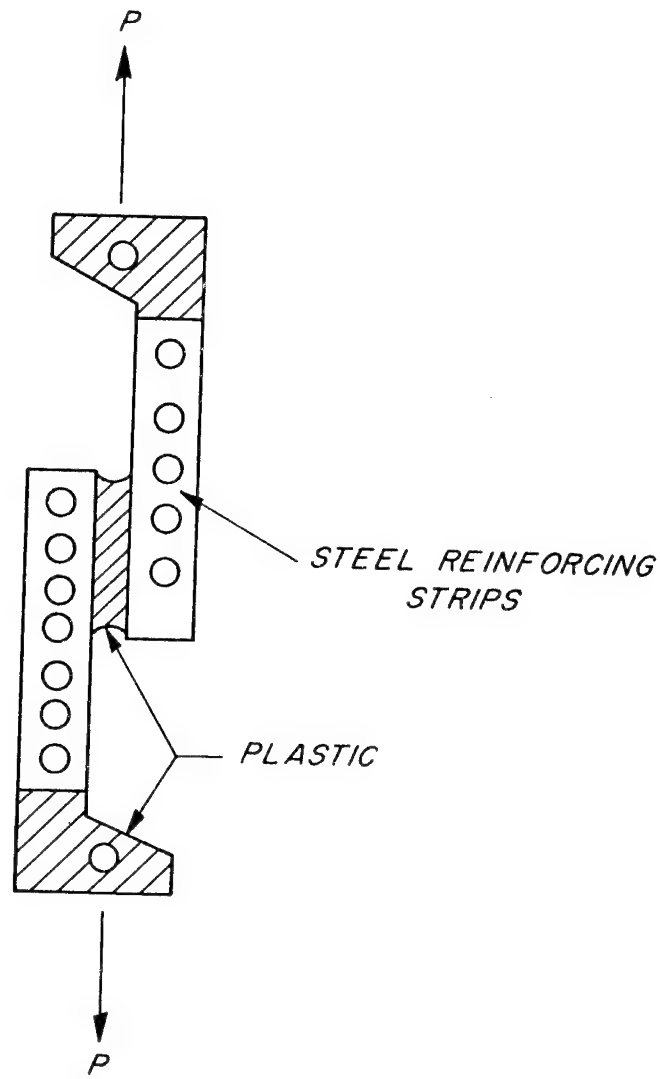


Figure 21. --Reinforced Monolithic Photoelastic Model Used by Mylonas (16).

Models were made using a mixture of ethoxylene resin, cyclohexanol and dibutyl-pthalate cast between 1/2-inch-square Bakelite bars reinforced with stainless steel studdings. The shape of the air-adhesive interface was controlled by either machine shaping it or using soluble-salt forms that could be dissolved away from the model after the resin had cured. The adhesive in the model was a block of resin 1/2 by 1/2 inch square and of varying lengths. Mylonas points out that this square adhesive gives rise to a complex three-dimensional stress distribution in the model, while in an actual joint it is one of plane strain.

The purpose of the study was to study the effect of the adherend-air interface on the stress distribution at the end of the joint. Tests were conducted first to determine whether the length of overlap had any effect on the stress distribution. It was concluded that as long as L/t was larger than 3, the length of overlap had no effect.

It was found that for curved air-adhesive interfaces (refer to Figure 22) the stress varied from tnesion to compression along the interface, with an isotropic point in the center. As the radius of the interface was changed, the highest stress was found to be on the air-adhesive interface away from the adherend for the smallest radii; for larger air-adhesive interface radii, the highest stress moved closer to the adherend-adhesive interface. It was concluded that for small radii the joint was most likely to fail cohesively in the adhesive, and for large radii the joint would fail along the adherend-adhesive interface. Refer to Figure 22.

For models with straight air-adhesive interfaces (Fig. 22) it was found that the maximum stress concentration in the adhesive was greatest for a 90° angle of inclination between the interface and the adherend, and that the stress concentration decreased as the angle decreased.

Data was also presented on the stress distribution in butt-type joints.

Mylonas has shown that the geometry of the air-adhesive interface has a strong effect on the stress distribution in the joint. The only questions concerning the study are whether these large 1/2-inch-square blocks of resin behave the same as a much thinner adhesive film, and whether these models are similar to an adhesive joint.

<u>Demarkles, L. R.</u> (<u>18</u>)

Investigation of the Use of a Rubber Analog in the Study of Stress Distribution in Riveted and Cemented Joints. NACA Technical Note 3413, November 1955.

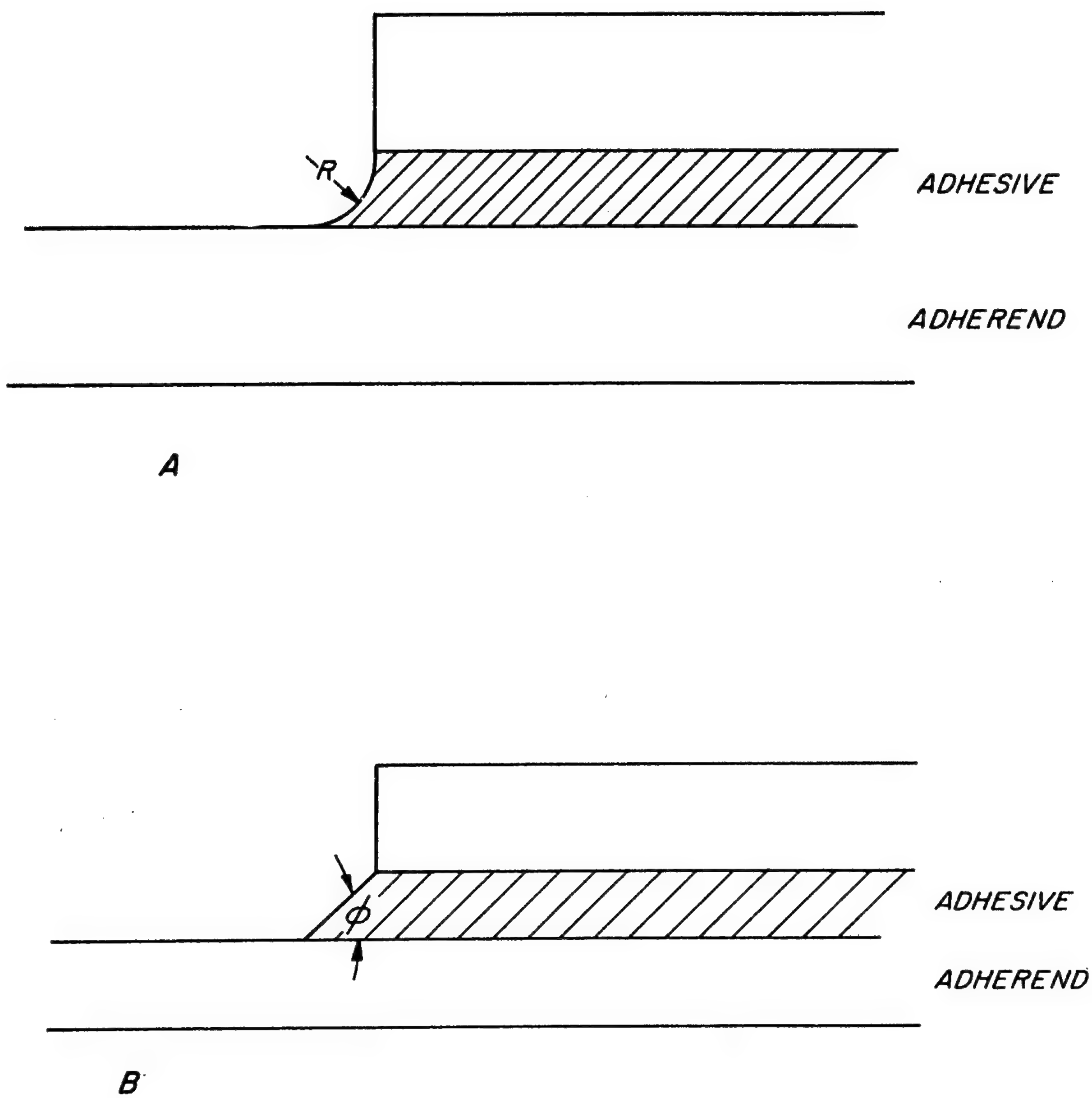


Figure 22.--Two Typical Air-Adhesive Interfaces in Lap Joints: A, Curved; B, Linear.

An investigation was made of the stress distribution on models of riveted and bonded lap-type joints made from foam rubber. The deformations in the models were exaggerated due to the low modulus of the rubber material and it was possible to actually experimentally determine the deformations.

Models of bonded joints were made by simply bonding together two sheets of foam rubber using a latex adhesive. The adhesive used had the same mechanical properties as the foam rubber. The model then was analogous to a joint in which the adhesive and adherends had the same properties and the adhesive was infinitely thin. The same model could probably have been obtained by simply cutting the entire joint from a single piece of foam rubber.

Deformation measurements were made on the joint by photographing the displacements occurring in a reference grid painted on the specimen using black shellac. Shear stress and normal stress distributions were determined from the deformations. These distribution curves are actually the stress distribution in the adherends, and not in the adhesive. In these models there is essentially no adhesive present and the models do not really simulate a lap joint, but rather an offset in a solid piece of material.

An attempt was made to determine the stress distribution on an actual joint by placing a photogrid with a 0.01 inch spacing on a Redux-bonded magnesium lap joint. The photogrid was applied in the area of the ends of the overlap. The joints were loaded to failure. The deformations in the joint were too small to be indicated by any change in the photogrid.

An analysis of a lap joint was presented that was similar to the Volkersen analysis of a supported lap joint without bonding. A comparison was made of the shear-stress distribution along the lap predicted by the analysis and that obtained from the rubber models, and the theoretical analysis predicted higher stresses than the models.

McLaren, A. S., and MacInnes, I. (46)

The Influence on the Stress Distribution in an Adhesive Lap Joint of Bending of the Adhering Sheets. British Journal of Applied Physics, 9:72-77, 1958.

A study was conducted to determine the effect of the bending moment on the stress distribution in models of lap-type joints using conventional-photoelastic stress analysis. Two types of models were used, a homogeneous plastic-sheet model and a composite model consisting of two types of plastic bonded in a manner to simulate a lap joint. Different degrees of bending were obtained by applying a tensile load at different angles to the line of the joint. The effect on the stress distribution of the shape of the air-adhesive interface at the end of the overlap was also studied.

In the homogeneous joints cut from a single plastic sheet, the adherends were 0.5 inch thick, and the adhesive thickness varied from 0 to 0.5 inch. In the composite models, the adherend and adhesive thickness (t) were both 0.5 inch. The modulus of elasticity of the plastic used to simulate the adherends was 20 times that of the adhesive. Refer to Figure 23.

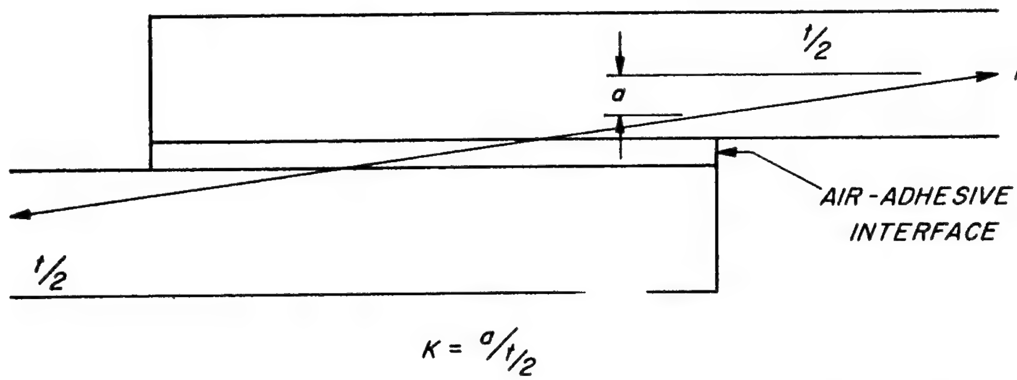


Figure 23.--Bending Moment Factor Affecting Stress Distribution in Lap-Type Joints.

The bending moment was measured as a function of the moment factor K, where $K = \frac{a}{t/2}$. "t/2" is one-half the thickness of the adherends, and represents the point through which the line of force will pass in the adherends when the joint area has reached its maximum rotation. "a" is the distance from the centerline of the adherend to the line of the applied force, just as the load is being applied. "a" is determined at a position in the adherend just at the end of the length of overlap. This is shown in Figure 23. For $K > 0$, the joint rotates in a conventional manner. For $K < 0$, the joint area is actually given a negative moment to overcome the positive moment due to the offset in the lap-type joint.

For both the homogeneous and composite models for $K < 0$, it was found that the maximum isochromatic fringes occurred at the ends of the length of overlap and then decreased toward the center. For large positive K values, the fringe order increased proportionately at the ends of the overlap, but remained relatively unchanged near the center of the joint.

For $K > 0$, it was found that the opposite relationship occurred. It was possible to almost eliminate the stress concentration at the ends of the overlap and to have the stress distribution increase toward the center of the overlap. This occurred in both types of models.

An indication of the difference between homogeneous and composite models was given by the results when K = 1. This is the condition when the line of the applied load passes through the centerline of the adherend at the end of the bonded area. In the homogeneous model, it was found that the directions of

the principal stresses were parallel and perpendicular to the line of the joint both in the adherends and the adhesive, except at the very ends of the length of overlap. This means there was no shear transfer of load across the bond line along the central area of the joint. In the composite model, the stress trajectories were perpendicular and parallel to the line of the joint in the adherends, but were at 45° angles in the adhesive. This indicates that the adhesive was mainly in longitudinal shear throughout its length, except at the very ends. This would support the premise that joint studies made on homogeneous models do not truly simulate actual joints, except perhaps for a bonded joint where the adherends and adhesive are exactly matched in mechanical properties. This condition is difficult to visualize in actual practice.

The results of studies on the effect of the shape of the air-adhesive interface at the end of the joint were generally the same as those obtained by Mylonas (49).

This study has added further clarification to the effect of the bending moment in the lap-type joint.

Hahn, K. F. (27)

Photostress Investigation of Bonded Lap Joints; Part II, Analysis of Experimental Data. Douglas-Aircraft Company Research Report SM 4000-1, 1960.

The stress distribution was studied in the adherends of lap-type joints using a reflection photoelastic-analysis technique. The joints studied were of 0.250-inch-thick aluminum sheets bonded with Metlbond 4021. The total length of the joint was over 16 inches, the adherend width 2 inches, and the bonded area 2 by 2 inches. A homogeneous joint of solid aluminum was cut to a similar geometry.

Photoelastic plastic was applied to the two large joint surfaces, and the stress distribution was determined in the adherend along the centerline of the adherend as a function of the distance from the end of the bonded area. A correction factor was applied to the stress distribution to account for the increased stiffness and shift of the neutral axis in the adherends due to the photoelastic plastic bonded to the adherend surfaces.

The stress distribution was found to be a maximum in the adherend at the edge of the bonded area and then decreased with the perpendicular distance from the bond. The results agreed very well with those predicted by the analysis developed in Part I of the report. Within the bonded area, the stress was found to be a maximum slightly inside the length of overlap, then decreasing toward the center of the bonded area.

The test results also indicated an effect heretofore not considered in relation to joints. The experimental stress distribution was found to be nonuniform across the width of the joint. It was found that stress concentration occurred at the edges of the joints near the reentrant corners of the bond area. This was attributed to the Poisson's effect, which causes an antielastic curvature in the adherend as it is being deformed by the bending moment. This nonuniform distribution across the joint width was also noted within the bonded area. One specimen was loaded to a point that was calculated to cause yielding in the adherends, but no change in the stress distribution was observed.

This reflection photoelastic technique appears to be one possibility for studying stress distributions within the center of a bonded area, particularly if the adherends are not thick. It may be possible to learn something of the effect of bond continuity on the stress distribution.

Kutscha, D. (37)

> Photoelastic Analysis of Shear Stress Distribution in Adhesive-Bonded Lap Joints. U.S. Forest Prod. Lab. Report TP-122, July 1962.

A study was conducted to determine the effect of the length of overlap on the shear stress distribution in an adhesive-bonded lap joint and compare the distribution obtained with that preducted by the Goland and Reissner (24) analysis. The study was made on actual joints using a photoelastic adhesive. Transmission photoelasticity was used to determine the stress distribution on the exposed edge of the adhesive film.

The joints were 0.064-inch-aluminum alloy, 0.25 inch wide, bonded with Photostress A at a film thickness of 0.029 inch. The length of overlap varied from 0.28 inch to 1.03 inches. The ratio of the shear modulus of the aluminum to that of the adhesive was 22:1.

The shear stress was found to change from a minimum at the center of the joint to a maximum at the edge of the overlap. For a given average shear stress, the maximum stress-concentration factor varied from 1.75 at 0.35 inch to 3.5 at 1.03-inch length of overlap. In all instances, the experimental stresses were higher than those predicted by the Goland and Reissner analysis. It appeared as though the adhesive was behaving as a stiffer material in the joint than it did in its bulk form. This was not further verified because an actual determination of the modulus of the adhesive in the joint was not made. The value for the shear modulus used in the calculations was that which had been determined for bulk material. The edge of the length of overlap showed the most complex stress distribution, as previously shown by Mylonas (49) and McLaren and MacInnes (46), and it is believed that more work should be done in this area.

This preliminary study appears to be the closest approach to determining the stress distribution on a film in an actual joint.

IV. MECHANICAL PROPERTIES OF ADHESIVE FILMS WITHIN JOINTS

A. Adhesive Film - Tests Within Joints

The mechanical properties of an adhesive film within a joint is the one area of research which has received the least amount of attention in the general area of mechanics of adhesive joints. The importance of this topic cannot be ignored since in all the analytical work one of the independent variables is the modulus of rigidity of the adhesive. It is immaterial how sophisticated an analysis may be, if one does not have the materials-property data to substitute in the analysis. The same point can be made concerning the material properties of the adherends, but in most cases this information is available.

The problem of determining the properties of an adhesive film while it is actually in place in a joint has received little attention thus far. It is a difficult problem to approach experimentally for the same reasons that determining the stress distribution in a joint experimentally is difficult. Primarily, the amount of material available for experimental manipulation is very small. Related to this is the problem of having a proper specimen that will give the desired stress condition in the adhesive.

The question was raised earlier as to why is it not possible to use mechanical property data determined either on bulk material or free film material, rather than on thin-film material in a joint. Several reasons could be proposed:

(1) Adherend-mechanical restraint.

When an adhesive polymer film is continuously bonded throughout a joint it is offered a certain amount of restraint by the adherends. In some instances this may be thought of as adherend support. If the adhesive undergoes a volume shrinkage during cure, the adherends do not allow this to occur freely throughout the adhesive film, and in some instances this restraint may be non-uniform. This condition could affect the modulus of rigidity of the adhesive by introducing some preferred orientation into the polymer and would affect the strength of the film by introducing some pre-stress into the film or even some micro-cracks or discontinuities if the stresses are large enough. This question of adherend-restraint is probably the major factor affecting data obtained on films insitu, bulk material, or free films. When the film is loaded insitu, the internal stress distribution is likely to be different than when it is loaded in the unrestrained state.

(2) Adherend - chemical affects.

It is a well-known fact that some adhesives will bond to one adherend but not at all to another. In other instances the adhesive will bond satisfactorily to two different adherends but one bond will be stronger than another. It is difficult to say exactly how much of this is due to the initial wetting of the adhesive on the adherend, and how much is due to the adherend actually influencing the geometry of the polymer molecules at the interface, with a subsequent change in mechanical properties. One must be cautious when considering a property such as strength and then attempting to relate this to modulus of rigidity. These properties are not based on the same material parameters, and something that could affect the strength of the joint might not affect the elastic or visco-elastic properties of the adhesive film. The modulus of the adhesive would be affected by the basic chemical structure and molecular arrangement of the polymer in the film, while the strength would be determined by residual stresses and film discontinuities.

(3) Bulk material effects.

The strength of a material is directly related to the size of the specimen tested. The classic example of this is the increase in strength of metals in whisker form compared to conventional test specimens. This phenomenon is normally explained on the basis that in a large specimen the statistical possibility of finding a discontinuity or flaw, which will affect the strength, is much larger than in a small specimen. This same phenomenon is demonstrated by adhesive joints. Usually there is some minimum adhesive-film thickness at which the joint exhibits its maximum strength in lap joints (50). Film thicknesses larger than this minimum exhibit reduced strengths. One would then expect that a strength test of some bulk adhesive would yield lower strengths than that of a thin film, because of a higher probability of a presence of flaws.

The question of material restrain would also apply to shrinkage or cure of an adhesive in bulk form. One would expect that adhesive material in bulk form would not be subjected to the restraint experienced during cure within a joint.

All of these questions and possible answers are only hypothetical since none have been extensively investigated. If the questions could be adequately answered, it may become possible to predict the behavior of an adhesive from some tests conducted with a more manageable bulk-type specimen.

B. Criteria for Specimen Design

To provide the proper data to satisfy an analysis of the elastic stress distribution, it is necessary to obtain two of the following three material constants for the adhesive film: modulus of elasticity (E_a), modulus of rigidity (G_a), or Poisson's ratio (μ). Of these three, the modulus of rigidity is of the most interest, since in almost all adhesive applications the adhesive is loaded in shear. Under pure-shear loading ($\underline{E}$) and ($\underline{\mu}$) are not important, since Poisson's effect does not occur. The requirements, then, for a specimen to determine this material property are as follows:

(1) The adhesive film should be subjected to a simple shear load only. There should be no tensile or peeling forces in a direction perpendicular or parallel to the plane of the adhesive.

(2) The shear distribution should be uniform throughout the adhesive film.

(3) The specimen should be simple, inexpensive, and easily made and tested. Thus far, no specimen is available which adequately meets these requirements for all situations of adherend-adhesive combinations. The requirements are almost self evident, but their importance cannot be overemphasized. It is only through emphasis on points such as these that it will ever be possible to eliminate the huge amount of empirical, time-consuming, and costly testing that occurs whenever some new adhesive or adhesive application becomes available. This type of information coupled with a reliable method of analysis would do much towards advancing the more efficient and scientific use of adhesives.

Another point not to be overlooked is that information concerning the simple mechanical properties, uncomplicated by complex stress fields, would aid in the evaluation of new adhesives and the effect of various material-composition parameters on mechanical behavior. It would be more meaningful to study the effect of increased molecular weight of the adhesive polymer on the elastic shear modulus than on the lap-shear strength, for the latter is complicated by a non-uniform stress distribution of complex stress combinations, extending from the elastic to visco-elastic range of material behavior, all at the same time.

This brings up the point of materials behavior beyond the simple isotropic, elastic case. Almost all polymers fall into the class of visco-elastic materials. Their mechanical behavior is dependent on the stress, rate of stress application, time and temperature. In many cases the tensile-compressive stresses and shear stresses relax at different rates; therefore, complicated stress relaxation occurs especially under combined stress conditions. Since these relaxation times are temperature dependent if any

thermal gradients occur in the specimens, the situation becomes even more complex. To completely describe the behavior of a single polymer system the undertaking becomes truly formidable. For practical purposes, it would be necessary to specify the range of temperatures, stress levels and stress combinations very critically to make the problem amenable to solution. This problem is not unique to adhesives, but is occurring wherever organic structural materials are in use. Before this problem is approached from the most complicated form possible, it would be helpful if it could be determined that the adhesives have a true elastic modulus.

C. Torsion-Type Specimens

The specimen type most readily available as a means of applying a uniform, simple-shear stress to a material is some form of torsion specimen. Debruyne (50) suggested a metal to metal specimen of this type. The specimen consisted of two thin-walled cylinders bonded end to end and then subjected to a torsional load. A form of this specimen is shown in Figure 24.

The shear stress applied to the adhesive is calculated from the applied torque and cylinder diameters.

$$\tau = \frac{16\, d_1 T}{(d_1^4 - d_2^4)} \tag{63}$$

If the outside diameter of the tube (d_1) is large with respect to the wall thickness of the tube or the width of the bonded area $\left(\frac{d_1 - d_2}{2}\right)$, essentially there will be a uniform shear distribution across the width of the joint, and the adhesive will be subjected to uniform simple shear throughout the entire joint.

The shear deformation in the adhesive can be determined by noting the relative movement of the two cylinders; this can be accomplished by placing two reference marks on opposite sides of the bond line and measuring their movement with a microscope or suitable optical strain gage with a long optical lever arm. The shear strain is then calculated knowing the deformation and bond line thickness.

$$\gamma = \frac{\delta}{t_a} \tag{64}$$

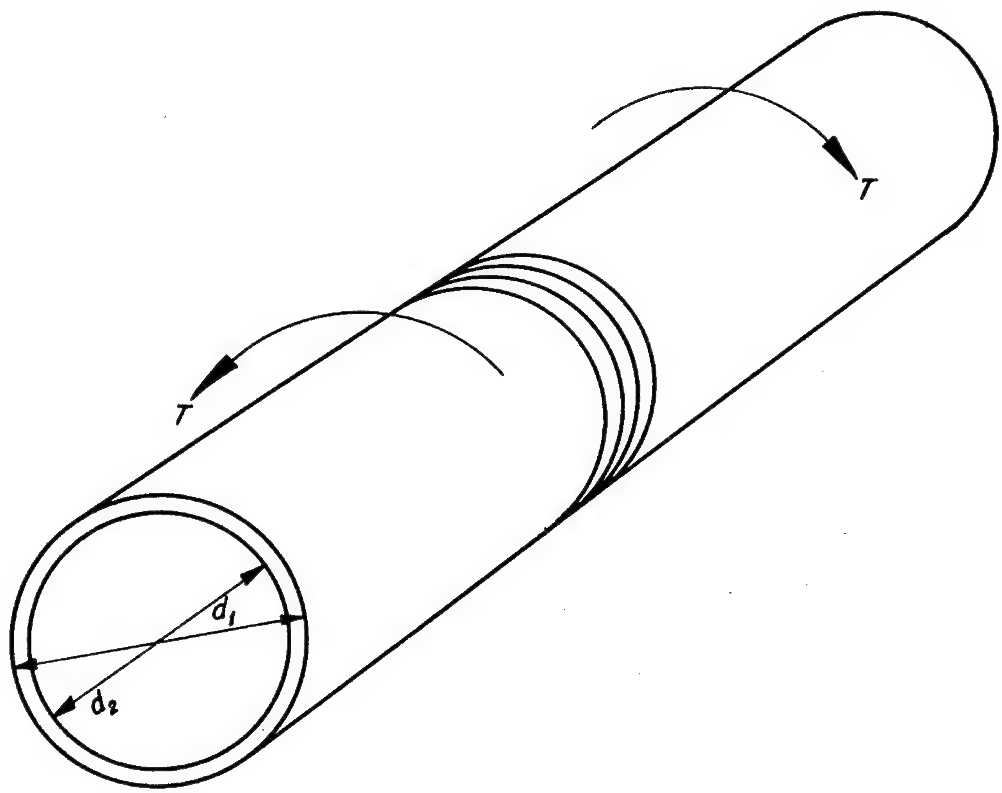


Figure 24. --Cylindrical Torsion Specimen With Four Bond Lines.

The elastic modulus of rigidity is then calculated from

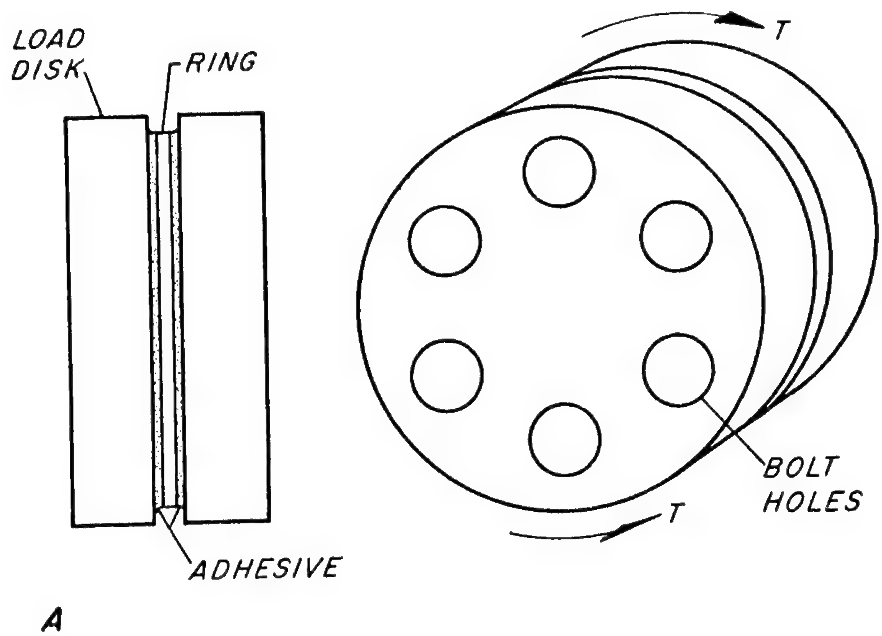
$$G_a = \frac{\tau}{\gamma} \tag{65}$$

Because shear strain (γ) is directly related to the adhesive film thickness it is very important to obtain an accurate measurement of this film thickness. When aluminum or aluminum-alloy tubes are used, and there is any machining done on the specimens after they are bonded, there is a tendency for the soft aluminum to flow into the bonded area and obscure the bond line. It then becomes difficult to obtain an accurate measurement of film thickness and to determine what the variation in this film thickness might be. An accurate measurement of the film thickness is just as important as an accurate measurement of the shear stress and strain.

Tooley (62) used a modification of the torsion specimen in which he used flat circular disks that had been machined on one side so that a 1/8-inch or 1/4-inch lip projected from one face of the disk along its circumference (Fig. 25 B). Two disks were then bonded together lip to lip. Deflection measurements were made with a microscope. A modulus of rigidity was determined for three adhesives: Metlbond MN3C, Metlbond 4021, and FM-47. These are given in Table I.

Another modification of the torsion test was used by Kuenzi (35). Tests were made using the cylinder-torsion specimen but with both single and multiple bondlines. The multiple bondlines were obtained by bonding together several short sections or rings cut off the same cylinders used for the ends of the specimen. Up to 10 bondlines were tested at once by bonding together 9 rings between two longer end-sections. The multiple bondline specimens were used for obtaining the modulus of rigidity of particularly stiff adhesives in which the shear deformations were relatively small. The adhesive deformations were determined by measuring the total deformation across all the bondlines and subtracting the computed deformation for the metal rings between the bonds. Modulus of rigidity data was determined for: Redux K-6, Scotchweld AF-6, Metlbond MN3C, and Epon VIII, and is given in Table I.

Gillespie and Rideal (23) used the cylinder-torsion test with a single bond to study the visco-elastic behavior of paraffin wax and cellulose-nitrate adhesives bonded to brass and steel. Some of the difficulties in testing adhesive bonds even under simple shear are pointed out by the fact that great difficulty was encountered in preparing specimens that would behave in the same manner mechanically. Some of the variation was attributed to inaccuracies in measuring film thickness and to a variation in film thickness throughout a joint.



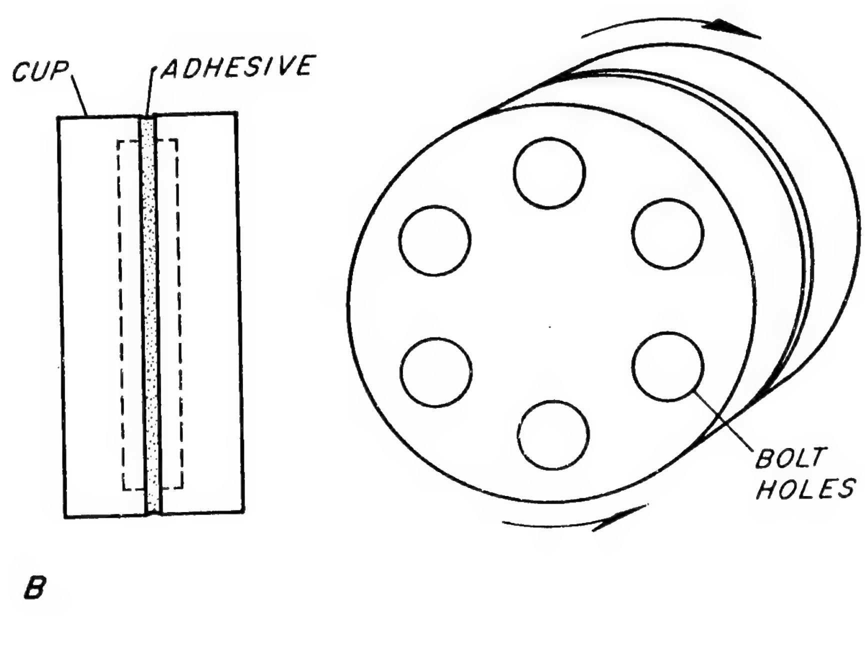


Figure 25.--Cylindrical Torsion Specimens; A, Disk Ring Type; B, Cup Type.

Lunsford (43) used another variation of the torsion test by simply bonding a metal ring between two circular disks and then measuring the shear deformation from the relative movement of the outer disks. (Fig. 25 A). This arrangement involves two bondlines. Data was obtained for the modulus of rigidity of FM-47 and is given in Table I. Several specimens of this type and the general testing arrangement were inspected during a visit to General Dynamics Corp. in February 1963. At this time modifications had been made on the specimen so that it was possible to do testing at different temperatures. This was accomplished by hollowing out the two end shear-plates and pumping heat-transfer fluids through the plates. It was then possible to determine shear modulus, creep, and relaxation as a function of temperature.

A further study by Lunsford (45) using the plate-ring torsion specimen, presented data for two other adhesives: AF-31 and Plastilock 620, which is given in Table I.

Kuenzi and Stevens (36) presented data for five more adhesives: Metlbond 4021, FM-47, Epon 422J, Metlbond 408, and FM-1000. Data on these materials was obtained using the cylinder torsion specimen with both single and multiple bondlines. This report summarizes the early work by Kuenzi and represents probably the largest amount of published information concerning modulus of rigidity, pure shear strength, and modulus of elasticity of thin films of adhesives.

Enough data is presented to indicate that although a simple shear specimen of the torsion type should provide more meaningful information than the more complicated simple-lap specimen, a certain scatter of shear modulus values is still obtained. For some adhesives there was an apparent relationship between shear modulus and total adhesive film thickness in a multiple bond-line specimen. The shear modulus of FM-47 appeared to be directly proportional to the film thickness. It is not believed that this variation is significant, but is rather experimental scatter of the data. Where the shear modulus is reported for FM-47 as 117,000 psi, the variation was from 92,000 psi to 147,000 psi for a change in film thickness of from 0.003 to 0.005 inch. No specification was made as to what the rate of load or rate of strain application was for the specimens. This could have a marked effect on the behavior of the adhesive.

Some form of the shear-torsion specimen should be used in an extensive testing program in which very close control is maintained over all the bonding and testing variables to obtain a good evaluation of this test for elastic properties.

D. Lap-Type Specimens

Several attempts have been made to use either deformation data or joint strengths from the simple-lap specimen to obtain the elastic properties of an adhesive. If one could assume a uniform shear distribution in the adhesive and that no normal or peel stresses were present, it would be possible to relate lap-joint deformation to the applied load and determine the shear modulus. There is a joint condition in which these requirements are met. These criteria are satisfied by a double-lap joint in which the adherends are thick and their modulus of rigidity is very large compared to that of the adhesive. In this instance the adherends would undergo essentially rigid-body displacements, consequently subjecting a relatively thick film of a low-modulus adhesive to pure shear.

This technique was used by Goodyear Aircraft (6) to determine the modulus of rubberlike adhesives which had shear modulus values of approximately 100-150 psi. Double-lap joints were made using 1/8-inch-steel adherends and adhesive-film thickness of 0.005 to 0.030 inch. Shear strains were obtained by measuring the relative displacements of the adherends. This technique appears to be suitable for materials of this type.

The other method of using lap-joint data to determine material properties is derived from the analysis of the simple, single overlap joint subjected to shear load only. Using the Volkersen analysis for a joint with adherends of the same thickness and with the same properties, one obtains the following relationship for the maximum shear stress in the adhesive:

$$\frac{\tau_m}{\tau_a} = \Delta \coth \Delta. \tag{66}$$

where

$$\Delta^2 = \frac{G a L^2}{2 E t_1 t_2} \tag{67}$$

Assuming that the adhesive will fail in shear and the maximum shear occurs at the edge of the lap, one can plot failure shear-stress of a joint as a function of length of overlap and extrapolate this curve to zero overlap, to obtain the shear strength (τm) of the adhesive. Knowing (τm) one can plot a curve of τm τa vs. Δ using equation (1) and suitable values for Δ.

A suitable range is $\Delta = 6.66$ to 0.4. With this curve and knowing τm one can obtain a Δ value for each joint tested.

Since

$$\Delta = \frac{L}{(t_1 t_a)^{1/2}} \qquad \left(\frac{G_a}{2E}\right)^{1/2} \tag{68}$$

one can plot Δ vs. $\dfrac{L}{(t_1 t_a)^{1/2}}$

and the slope of the straight line will be a measure of G_a. This method has been used by: Wan and Sherwin (65) to determine the shear modulus of Cycleweld; Broding (9) for Redux; and Eickner (20) for Redux K-6, Scotchweld AF-6, Metlbond MN3C, and Epon VIII. The shear modulus values are given in Table I. Exactly what the relationship of these effective-shear moduli is to true elastic-shear moduli is difficult to say.

The data used to calculate the shear modulus arise from strength tests in which the adhesives have exceeded the elastic limit. The first assumption made using this method is that adhesive should be subjected to shear loading only, but in all cases the strength data used was determined for lap joints allowed to rotate under load. In this instance the adhesive is in combined shear and tensile stress and probably does not even fail in shear, but in tension perpendicular to the plane of the joint.

Eickner (20) used the lap-joint method to compute shear modulus data from joints tested in a restrained and the unrestrained condition, and to determine whether the bending and subsequent peel stresses have any marked effect on the computed effective modulus. He found that for relatively stiff adhesives, like Redux K-6 and Epon VIII, the effective-shear modulus was higher if one used data for specimens unrestrained in bending; for less stiff adhesives, like Metlbond MN3C and Scotchweld AF-6, the effective shear modulus was lower when strength data from unrestrained specimens was used, compared to strength data from restrained specimens.

Eickner (20) had also used the Goland and Reissner analysis to compute effective shear moduli and found that this analysis predicted lower effective shear moduli for strength data for both restrained and unrestrained specimens than the Volkersen analysis. The Goland and Reissner analysis takes into account bending in the joint, and one would expect that it would predict higher rather than lower shear moduli than the Volkersen analysis.

An interesting comparison can be made using Table I which summarizes elastic and effective shear moduli as calculated by indirect methods for a variety of adhesives. An indication is given of how vastly different these values are for a particular adhesive. Note that for Epon VIII Kuenzi obtained 180,000 psi and

TABLE I. --MODULUS OF RIGIDITY OF SEVERAL ADHESIVES

Adhesive	Torsion joints					Lap joints		
	Tooley (62)	Kuenzi (55)	Lunsford (43)	Lunsford (45)	Kuenzi and Stevens (36)	Wan and Sherwin (65)	Broding (10)	Eickner (20)
	P.s.i.	P.s.i.	P.s.i.	P.s.i.	P.s.i.	P.s.i.	P.s.i.	P.s.i.
Cycleweld						2,680		
Epon VIII		180,000						26,311
Epon 422J					160,000			
Metlbond MN3C		1,530						8,026
Metlbond 4021	3,000				5,520			
Metlbond 408					49,300			
FM 47	150,000		154,600		117,000			
FM 1,000					64,100			
Redux							37,000	
Redux K-6		184,000						14,903
Scotchweld AF-6		6,100						5,097
AF 31				31,000				
Plastilock 620				6,850				

Eickner 14,903 psi; for Redux Broding obtained 37,000 psi; for Metlbond MN3C, the effective shear modulus was larger than the elastic modulus; and for Scotchweld AF-6 the effective modulus was smaller than the elastic modulus.

This comparison would indicate there is no real connection between these effective and elastic moduli. They both arise from completely different stress conditions.

E. Annotated Bibliography

Wan, C. C., and Sherwin, S. B. (65)

> Structural Characteristics of Bonded Metal-to-Metal Lap Joints. Chance Vought Aircraft Report MP 2020-2, June 1945.

This report presents an analysis of a rigidly supported lap-type joint and uses it to correlate strengths of aluminum joints bonded with a combination of Cycleweld and Durez adhesives.

It showed that the ratio of maximum shear stress to average shear stress in a joint can be expressed as

$$\frac{\tau_m}{\tau_a} = \Delta \coth \Delta \qquad (69)$$

where

$$\Delta^2 = L^2 \frac{Ga}{2Et_1t_a} \qquad (70)$$

A more complete discussion of this method of data treatment is given in the review of the work by Broding (10).

Using the analysis, a modulus of rigidity of 2,680 psi was calculated for Cycleweld. The stiffness of the Durez was neglected since it was approximately 200 times stiffer than the Cycleweld.

As part of this study a comparison was made of the relative strength of simple lap joints compared to semi-beveled lap joint one adheren was tapered or scarfed in thickness throughout the length of overlap, but it was not a complete taper down to a "feather edge". Strength tests of these joints bonded with the Cycleweld-Durez combination indicated they were not stronger than the simple lap joint.

Broding, W. C. (10)

Determination of Static Shear Strength of Redux for Design. Chance Vought Aircraft Report No. 7575, May 1952.

This report presents data on: (1) lap-shear strength, (2) maximum adhesive shear stress, (3) adhesive shear modulus, and (4) allowable-design shear stress for Redux adhesive in lap-type joints. Tests were run at -60°, 75°, and 200° F. The properties are calculated from lap-joint strength data using the following methods:

1. Lap-Shear Strength

Lap-shear strength was obtained by dividing the failing load by the bonded area of the joint.

2. Maximum Adhesive Shear Stress

Broding assumed that an adhesive lap joint will fail when the shear stress in the joint reached some limiting value. For uniform shear-stress distribution, the maximum shear stress (τm) would be equal to the average shear stress (τa) at failure.

It is assumed this would be true only as the overlap approaches zero; therefore, if τa is plotted as a function of L and then extrapolated to zero L the intercept is τm. Broding did this for Redux using different adherend thicknesses and obtained a family of parabolic curves which had a common intercept. The τm was also determined for different temperatures.

3. Adhesive Shear Modulus.

Broding used the analysis of a simple lap joint rigidly supported to obtain (Ga). This analysis lends to the following relation for the maximum shear stress:

$$\frac{\tau_m}{\tau_a} = \Delta \coth \Delta \tag{71}$$

where

$$\Delta^2 = L^2 \frac{Ga}{2\,Et_1t_a} \tag{72}$$

A graph of $\tau m / \tau a$ as a function of Δ was obtained by using equation (71) and choosing appropriate values of Δ. A suitable range is $\Delta = 6.66$ to 0.4. Using this curve and τm obtained in the first section, it was possible to obtain a Δ factor for each joint tested.

Since

$$\Delta = \frac{L}{(t_1 t_a)^{1/2}} \quad (\frac{G_a}{2E})^{1/2} \tag{73}$$

Δ was plotted as a function of $L\ (t_1 t_a)^{1/2}$ and the slope of the straight line was a measure of G_a. Broding obtained G_a for Redux at several temperatures. G_a varied from 12,000 psi at 200° F. to 240,000 psi at -60° F. At 75° F., G_a was 37,000 psi.

3. Design Curves

Design curves for lap joints were obtained by plotting average lap-shear strength as a function of $L/t^{1/2}$. Curves are given for each temperature. Using these curves, the necessary overlap-to-sheet thickness ratio can be determined to give a certain strength joint at a given temperature.

Curves are also given for strength of a lap joint as a function of the tensile stress in the sheet outside the lap joint.

The methods described in this report allow one to obtain an adhesive shear modulus but the modulus value is questionable since its calculation is based on ultimate strength properties of a joint.

Tooley, D. A. (62)

> Metal Adhesives Development Program. Appendix XII, pp. A175-A203, Determination of Shear Modulus of Adhesives. Convair Report FZM-364, Dec. 1954.

This report describes the initial work carried out at Convair to determine the modulus of rigidity of an adhesive film. Initially attempts were made to obtain load-deflection measurements on simple lap joints. These attempts were unsuccessful because of the complexity of the lap-type joint. Another attempt was then made using a torsion specimen as suggested by Mylonas and DeBruyne (50).

Specimens and a load fixture were built which simulated simple shear loading in the adhesive-bonded faces of a large diameter thin-walled tube. The specimens were 4-inch-O.D. disks of steel or aluminum which had 1/8-or 1/4-inch flanges projecting from one face along the outside diameter of the disks. The lips of two disks were bonded together. The disks were loaded by attaching bolts through holes bored through the inner parts of the disks (Fig. 25).

Deflection measurements were made by observing the relative movement of a scratch mark across the outside edges of the bonded disks with a microscope.

Film thickness of the bonded adhesive was obtained by microscopic measurement of the exposed bondline on the edge of the disks.

Stress-strain measurements were made and the shear modulus was calculated for the following adhesives:

	Ga
Metlbond MN3C	2,000 psi
Metlbond 4021	3,000 psi
FM-47	150,000 psi

One of the major sources of error found in these tests, and a probable cause of the data scatter, was believed to be errors in bondline measurement and possible variations in bondline thickness throughout a specimen. The calculated shear strains are extremely sensitive to the measurement of film thickness.

Eickner, H. (20)

Basic Shear Strength Properties of Metal-Bonding Adhesives as Determined by Lap-Joint Stress Formulas of Volkersen and Goland and Reissner. Forest Products Laboratory Report No. 1850, August 1955.

Lap-joint strength data was obtained for four different metal-bonding adhesives. Redux K-6, Scotchweld AF-6, Metlbond MN3C, and Epon VIII. The joints were tested in tension under a restrained condition and in a configuration in which they were allowed to bend. Computations similar to those used by Broding (10) were used to calculate the maximum shear stress in the adhesive and the shear modulus.

Using the Volkersen analysis for a rigidly supported lap joint:

$$\frac{\tau_m}{\tau_a} = \Delta \coth \Delta \tag{74}$$

and then the Goland and Reissner analysis for the unsupported lap joint,

$$\frac{\tau_m}{\tau_a} = \left(\frac{1 + 3A}{4}\right) 2 \Delta \coth 2\Delta + \frac{3(1-A)}{4}, \tag{75}$$

where

$$\Delta = L \left(\frac{G_a}{2Et_1t_a}\right)^{1/2}, \tag{76}$$

and A is a function of $\left(\frac{L^2\sigma_1}{t_1E_1}\right)^{1/2}$

The Goland and Reissner analysis in general was found to give higher maximum shear stresses than the Volkersen analysis. Curves comparing the two analyses are given.

A third method of expressing the lap-joint strengths was presented. This method is based on plotting joint strength as a function of $L^2\ t_1$ on a log-log graph.

Equations of the form

$$\sigma_1 = b\left(\left(\frac{L}{t_1}\right)\right)^M \tag{77}$$

were used to express the straight lines obtained.

In general, the Volkersen analysis gave effective modulus-of-rigidity values 1.4 to 2.4 times higher than the Goland and Reissner analysis, and in general the modulus of rigidity values calculated from tests of unrestrained joints were higher than those obtained for restrained joints.

<u>Kuenzi, E. W.</u> (<u>35</u>)

Determination of Mechanical Properties of Adhesives for Use in the Design of Bonded Joints. Forest Products Lab. Report No. 1851, Jan. 1956.

This report describes methods for obtaining the elastic-mechanical properties for adhesives in joints. Data is given for four adhesives: Redux K-6, Scotchweld AF-6, Metlbond MN3C Nylon tape, and Epon VIII.

A torsion-type specimen was used to obtain the data. The specimen consisted of two pieces of thin-walled tubing bonded together end-to-end. Strain measurements were made with a Tuckerman gage which measures the relative displacement of the tubes as they are loaded in torsion. For stiff adhesives in which the deformations were extremely small, multiple bond-lines were tested. These were obtained by bonding thin rings of the same tubing between the longer tubes. The overall-shear deformation was measured and then the adhesive deformation was obtained by subtracting the calculated strain in the rings.

The film thickness of the cured adhesive was obtained by measuring the difference between the length of the tubes plus rings, before and after bonding. For multiple gluelines, only an average bond thickness can be obtained with this technique.

The modulus of rigidity of the adhesives was obtained by loading the tubes in torsion which applied a simple shear load to the film without any complicating tensile loads. The modulus of elasticity was obtained by loading the specimen either in compression or tension parallel to the axis of the tubes. Knowing Ga and Ea, Poisson's ratio was calculated from these values using the usual relationship for isotropic materials.

A summary of representative shear modulus data is as follows:

	Ga
Redux K-6	184,000 psi
Scotchweld AF-6	6,100 psi
Metlbond MN3C	1,530 psi
Epon VIII	180,000 psi

Using these experimentally determined modulus values, graphs were plotted showing the shear stress and normal stress distribution for a representative lap-joint geometry, bonded with a stiff adhesive and a flexible adhesive. The data was used with the Goland and Reissner analysis.

This report is the most extensive collection of data available concerning the mechanical properties of adhesives within joints. Although the data presented is on the elastic properties of adhesives, it should also be possible to apply the testing techniques described to cover the range of inelastic behavior.

Gillespie, T., and Rideal, E. (23)

The Deformation and Strength of Napkin Ring Metal-Adhesive-Metal Joints. J. Coll. Sci. 11, pp. 732-747 (1956).

This was a study of the mechanical properties of paraffin wax and cellulose-nitrate adhesives bonded to brass and steel. A torsion-type specimen was used that consisted of two pieces of thin-walled tubing bonded end-to-end. The tubes were loaded in torsion and the deformation of the adhesive was studied as a function of load, rate of load, and time. Film thicknesses of the adhesives were in the range of 0.005 to 0.006 inch. It was very difficult to make joints with reproducible visco-elastic properties.

Several general observations were made. For the wax-metal joints the shear strength was highly variable. It appeared to be affected by the previous load history of the joint and usually the joints that deformed the most were the weakest.

For the cellulose-nitrate brass joints there was also a wide variation in strength, and this variation increased with film thickness. Contrary to the wax joints, the cellulose-nitrate joints which deformed the most gave the highest strengths. It was also found that the shear modulus of the cellulose nitrate was stress dependent. The creep and nonrecoverable deformation were also discussed.

A conclusion drawn from the study was that the failure of a joint appears to be intimately related to the rupture of the adhesive, and to sort out the factors peculiar to adhesion at the interface it will be necessary to know more about the adhesives themselves at high stress.

Lunsford, L. R. (43)

Bonded Metal-to-Metal Shear Testing. ASTM Sp. Tech. Publ. 289, pp. 46-56, 1960.

A general review was presented of metal-to-metal bonding and of the factors which affect joint strength. These factors included manufacturing variables and joint-design factors such as joint geometry and the mechanical properties of adherends and adhesives. Representative data are given for several adhesives tested as single-lap joints, double-lap joints, as a composite I-beam specimen in which the adhesive lies at the neutral plane of the beam, and as a torsion shear specimen.

The torsion shear specimen gave the following shear-modulus values for the FM-47 adhesive:

Specimen	Ga
FM 47-65	168,300 psi
FM 47-66	106,900 psi
FM 47-67	188,600 psi

A theoretical analysis was presented for the stress distribution in an adhesive used to bond the facings in a sandwich panel to a thicker edge member. The problem is treated as a lap joint subjected to differential straining and a result similar to that of Volkersen (64) was obtained for the maximum shear stress in the adhesive:

$$\tau_m = \frac{G_a \sigma_1}{E_1 t_a K} \tanh K L \qquad (78)$$

$$K = \left[\frac{G}{E t_a} \left(\frac{1}{t_1} + \frac{1}{t_2}\right)\right]^{1/2} \qquad (79)$$

The importance of obtaining shear modulus values for adhesives, to use in analyses such as these, was stressed.

Hahn, K. F. (29)

Lap Shear and Creep Performance in Metal-to-Metal Bonds. Adhesive Age 4, No. 12, p. 34-39, 1961.

This paper presents information on the lap-shear strength of several adhesives as a function of temperature, the creep of the lap joints, and the modulus of elasticity of some free films of the adhesives at several temperatures. The free-film behavior is used to explain some of the behavior of the joints.

The adhesives discussed are primarily ones used by the German metal-bonding industry, but the free-film data presented may be of interest for comparison with material properties obtained on adhesives in joints.

Lunsford, L. R. (45)

Adhesive Torsional-Shear Test, General Dynamics--Ft. Worth, Report ERR FW-134 (Structures), Feb. 1962.

This report presented a full description of the adhesive torsional-shear test used by General Dynamics for determining the shear strength and modulus of rigidity of adhesives. The specimen and load device are modified versions of those developed earlier and reported on by Tooley (62).

The specimen consists of metal rings bonded between plane-circular disks (Fig. 25 A). The shear strain was obtained by measuring the relative movement of the two disks with an optical lever system. Mechanical property data were obtained for two adhesives. For one of the adhesives, AF-31, it was possible to detect the quality of the bond through variations in the shear modulus. The following representative data was obtained:

	Ga
AF-31 (nitrile rubber)	31,000 psi
Plastilock 620 (nitrile-rubber phenolic)	6,850 psi

Kuenzi, E. W., and Stevens G. H. (36)

Determination of Mechanical Properties of Adhesives for Use in the Design of Bonded Joints. U.S. Forest Service Research Note FPL-011, Sept. 1963.

This report was a revision of an earlier work by Kuenzi in which a torsional shear specimen was used to determine the shear modulus and modulus of elasticity of an adhesive film within a joint. The same technique was used in this work to determine the properties of five more metal-bonding adhesives. The data obtained for shear modulus was as follows:

	Ga
Metlbond 4021	5,520 psi
FM-47	117,000 psi
Epon 422J	160,000 psi
Metlbond 408	49,300 psi
FM-1000	64,100 psi

V. FAILURE CRITERIA FOR LAP JOINTS

The most common criterion for the failure of a lap joint to serve its structural function is the fracture or complete destruction of the joint. This criteria applies both to bonded-structural assemblies and to routine lap-joint testing. The question to be answered is what are the factors that dictate how, where, and by what mechanism a given joint will fracture? In spite of the large amount of joint-strength data that have been obtained over the years, very little work has been conducted to determine the fracture criteria and mechanism for adhesive joints.

As pointed out in the introduction there are several major criteria used to explain fracture. They are based on the assumption that when a certain material property is exceeded at a given point the material will fracture. This implies two conditions: (1) a knowledge of ultimate properties of the material in question (shear strength), and (2) a knowledge of the causes for these factors to be exceeded in the material (stress concentration). This report is primarily concerned with the second condition, and a small amount of information has been located concerned with the first.

Lubkin (41) conducted a study to explain the strengths of adhesive scarf-type joints. He investigated which theory of failure best explained the joint strengths. The theories investigated were those based on: (a) tensile failure under tensile load, (b) shear failure, (c) octahedral shear failure, and (d) tensile failure under compressive load. The experimental data was best explained by the maximum principle-stress theory which assumes that the joint will fail whenever the tensile strength of the adhesive was exceeded at some point.

The other work in this area was that of Ripling, Mostovoy, and Patrick (Refer to Annotated Bibliography) at the end of this section. This study was based on the theory of fracture mechanics developed by Irwin. This assumes that fracture will occur in a material when the strain energy at a crack tip reaches a certain value. The material property governing this behavior was termed the strain energy-release rate G. When the critical value G_c is exceeded a crack will propagate . This paper was concerned with measuring G_c for an epoxy-type adhesive as a function of various joint parameters. Factors investigated were joint width, film thickness, strain rate, and type of crack propagation. In this study the cracks were initiated artificially and G_c was determined for maintaining crack motion.

There are two other factors involved in this area. These are crack initiation and crack arrest. It is important to know what conditions cause a crack to start from some imperfection or discontinuity in the material, and what can be done to arrest a crack perhaps with some built-in crack arrestors.

This area is so new to adhesives and to polymers in general that further work in almost any direction can be undertaken.

A. Annotated Bibliography

Lubkin, J. L. (41)

A Theory of Adhesive Scarf Joints, With Analysis of Test Results.
Phase I Report U.S. Naval Ordnance Contract Nord 13383, Sept. 1953.
Also Journal Applied Mechanics 24, No. 2, pp. 255-260, 1957.

An analysis was presented indicating that the stress distribution in the adhesive film in a flat-scarf joint was uniform for a joint with both adherends the same, and that the stress distribution was not affected by the scarf angle of the joint. It was also shown that for a flat-scarf joint in which the adherends were dissimilar there was a critical angle at which the stress distribution would be uniform. For the tubular-scarf joint it was shown that the stress distribution was uniform for similar adherends at any scarf angle, but for dissimilar adherends there was no scarf angle at which the distribution would be uniform.

It was initially assumed for the analysis that the adhesive had linear-elastic behavior, but since the analysis showed that a uniform stress distribution was obtained, this restriction was unnecessary and it was possible to state that the stress distribution would be independent of the mechanical properties of the adherends and adhesive. Also the adhesive was assumed to be relatively flexible in relation to the adherends, so this would be analogous to a metal-scarf joint bonded with an organic adhesive.

It was indicated that, since the adhesive stress distribution was uniform, the equations developed for shear and normal stress should hold for any stress-strain law and it should be possible to calculate failing loads for scarf joints. Relationships were then developed for various modes of failure in the adhesive as a function of scarf angle. These equations were for: (a) tensile failure for joints loaded in tension, (b) shear failure, (c) octahedral-shear failure, and (d) tensile failure for a joint loaded in compression.

The joint strengths predicted by the above relations were compared to actual test data, and it was found that the criteria of failure in principal tensile stress best fits the experimental data. This means that when the principal tensile stress in the adhesive reaches some limiting value, failure will occur.

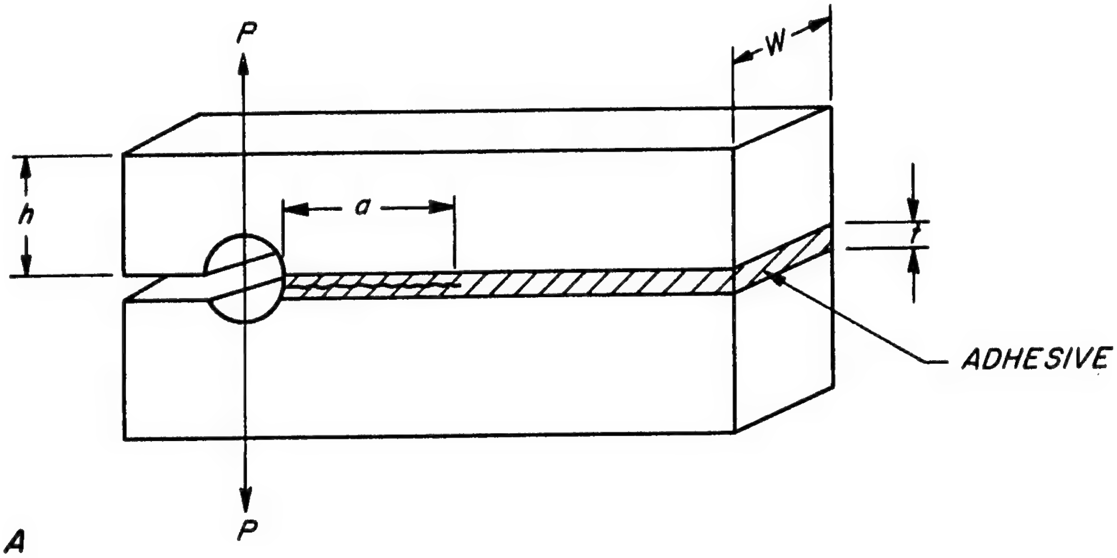
The procedure used in the study could be modified to cover criteria for failure of more complex form than those used by Lubkin.

Ripling, E. J., Mostovoy, S., and Patrick, R. L.

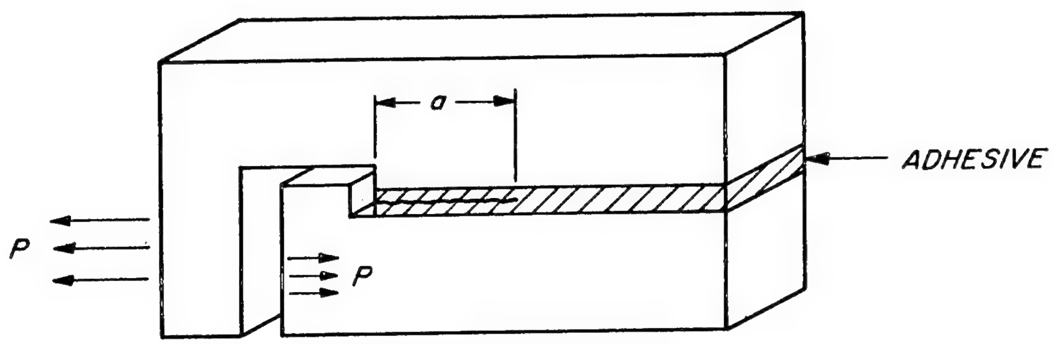
Application of Fracture Mechanics to Adhesive Joints. Materials Research Laboratory, Inc. Final Report Contract Nonr-3544(00)(x) Office of Naval Research, Also, Measuring Fracture Toughness of Adhesive Joints. Materials Research and Standards, 4, No. 3, p. 129-131, 1964.

This report covers the only work uncovered in which any of the newer concepts of fracture mechanics were applied to adhesive joints. The work is based on the fracture mechanics of materials developed by Irwin. For a full exposition of these concepts the reader is referred to G. R. Irwin, "Fracture Mechanics," in Structural Mechanics edited by Goodier and Hoff, Pergamon Press, 1960. Irwin's theory is based on the concept that cracking occurs in a material when the strain energy at the crack tip reaches a critical valve. This energy can be dissipated in any manner such as surface energy or irreversible flow. The amount of energy dissipated per unit area of crook extension is defined as the strain energy release rate (G) and the stress concentration at the crack tip (K). The magnitude for (G) and (K) at which a stationary or slow-moving crack will propagate are termed the critical values, (G_c) and (K_c). These parameters are material constants and are a measure of fracture toughness. The purpose of this study was to measure G_c and K_c for an adhesive joint as a function of several joint parameters.

Since adhesives are usually subjected to combined stress fields of shear and normal stresses G_c was measured under normal stresses where the stress was normal to the direction of crack propagation (G_{I_c}) and under shear stress parallel to the plane of crack propagation (G_{II_c}). The specimens used to determine these values are shown in Figure 26.



A



B

Figure 26.--Test Specimens Used in Measuring Strain Energy Release Rate During Crack Propagation (G): A, Opening Mode (G_{I_c}); B, Shear Mode (G_{II_c}).

The adhesive used in the study was an epoxy type. In order to establish some feeling for the behavior of the material, tests were run on bulk specimens prior to conducting the adhesive-joint tests. It was found that generally the bulk-material behavior paralleled that of the behavior in a joint, particularly that of the thicker bondline material.

In general three types of fracture surfaces were obtained. A peaked fracture associated with a fast rate of cracking (low toughness), a flat fracture as associated with slow propagation (high toughness), and a combination of both types. The effect of the various parameters was to change the type of the above fracture produced. The following results for G_{I_c} were obtained for the adhesive joints:

1. Joint Thickness--for a given joint width, as the film thickness increased, toughness was initially high, decreased to a minimum, and then increased again.

2. Joint Width--as joint width decreased from 1 inch to 0.25 inch the toughness generally increased.

3. Strain Rate--an increase in strain rate of 100 had a slight tendency to reduce toughness. In every instance the crack appeared to be moving at a higher rate than the load jaws were opening.

4. Crack Propagation--as a crack propagates, three values of G_{I_c} are involved. Energy must be expended to initiate motion of a stationary fracture, keep fracture propagating, and arrest fracture. The values determined in this work were associated with initiation of motion of a stationary fracture.

5. Shear Toughness-- shear toughness was of a higher order of magnitude than opening mode toughness. This was determined only for a limited number of tests.

This report represents the first work in the area of fracture mechanics of adhesive joints. It represents information on one particular material but the techniques developed could be easily extended to other types. In all instances in this work the cracks were artificially initiated with a razor blade, but actual adhesives would also contain flaws and discontinuities which would serve as crack sites. Exactly what the parameters of these crack sites must be to serve as a source from which a fracture could develop would be a study closely aligned to this work.

VI. EMPIRICAL METHODS OF JOINT DESIGN

The purpose of any empirical method of design is to provide some simple relationship between readily available experimental data and design criteria. This relationship can be based on a simple form of a more complex rational analysis, a large amount of experience with cut-and-try methods, or a best-estimate approach based on engineering judgment. In the case of lap joints a certain amount of each of these approaches has been used.

The ideal empirical method of joint design would allow one to design an adhesive-bonded structure using strength data from simple lap-joint tests. This information is experimentally simple to obtain and is readily available, since almost all adhesive users make tests of this type. It is a simple matter to obtain lap-joint strength as a function of adhesive-adherend combination, and variations in joint geometry. It becomes a matter then of expressing the joint strength as a function of some factor which will include the various independent variables.

The most suitable dependent variable is the joint strength or average shear strength of the adhesive (τ_a). This is chosen because it is usually the purpose of the adhesive to act as a shear-transfer medium in the structure, and the shear strength gives the best indication of this property. It is usually assumed that the adhesive is loaded only in shear. Using the method suggested by Tombach (61), we can express the empirical relationship functionally as:

$$\tau_a = \tau_m \cdot f_1 \left(L/t_a, L/t_1, L/t_2, t_1/E_1, t_2/E_2, E_a/E_1, E_a/E_2, G_a/E_1, G_a/E_2\right), \quad (80)$$

where f_1 and τ_m must be determined in some manner.

By making certain assumptions we can simplify this. Assume that we will be bonding one particular adherend of a certain thickness with one adhesive, always used at the same film thickness.

Then,

$$\tau_a = \tau_m \cdot f_2 \left(L/t_1,\ L/t_a\right), \quad (81)$$

where τ_m and f_2 must be determined in some manner. It can be further assumed that τ_a is independent of the overlap-to-film thickness ration and one obtains

$$\tau_a = \tau_m \cdot f_3 \left(L/t\right). \quad (82)$$

This equation represents the functional relationship most commonly used in the literature to correlate simple lap joint strength data. The lap-joint strength or failing stress $\tau_a = P/A$ is plotted as a function of (K), commonly called the joint factor, where K represents various forms of the overlap length to adherend thickness ratio.

$$\tau_a = f\left(K\right) \quad (83)$$

Some of the joint factors used are summarized as follows:

Author	K
Debruyne (16)	$\sqrt{t_1/L}$
Sheridan and Merriman (58)	L/t_1
Tombach (61)	$A(L/\sqrt{t_1})^B$
Brown (12)	$A + B(\sqrt{t_1}/L)$
Brown (12)	$\sqrt{\frac{t_1 + t_2}{2}} / L$
Brown (12)	$\sqrt{\frac{t_2(t_1+t_2)}{2t_1}} / L$
Brown (12)	$\frac{t_1 + t_2}{2} / L$

In the last study where Brown (12) was concerned with joints with different adherend thicknesses he found that a joint factor of $K = \frac{t_1 + t_2}{2} / L$ gave the best fit of the experimental data to a straight line.

In developing curves of the type described above, it is important to establish statistical criteria concerning the confidence limits of the data. All strength data has some form of distribution and it is important for the engineer to decide whether the strength data he has obtained on lap joints is representative of data that would be expected from production assemblies.

Once the $\tau_a = f(K)$ curves are obtained, certain criteria are set up concerning what ranges of the joint factor are to be considered acceptable for design purposes. For example, the Martin Company (5) has set the following limits:

Class 1. Best Quality - $L/t_1 < 60$

Class 2. Intermediate - $L/t_1 > 80$

In general when a design is chosen, L/t_1 should be at least 30 with at least a 1-inch length of overlap. For epoxy adhesives, the L/t_1 should be at least 100, where t_1 is for the thinnest adherend in a joint with different adherend thicknesses. The above criteria are just given as examples of factors used in a particular application.

The empirical method described above is the most common one used in the aircraft industry today. It can be related to a more complex rational analysis as shown by Mylonas and DeBruyne (50). Starting with the result obtained by Volkersen for a lap joint without bending,,

$$\frac{\tau_m}{\tau_a} = \Delta \coth \Delta. \tag{84}$$

Note that

$$\tau_a = \tau_m \cdot f(\Delta), \tag{85}$$

where

$$\Delta^2 = \frac{G_a L^2}{E_1 t_a t_1}. \tag{86}$$

For a series of joints then bonded with the same adhesive, Δ is a function of L/t_1 and therefore

$$\tau_a = f\ (L/t_1) \tag{87}$$

which is same result obtained previously.

There are several major disadvantages associated with the use of these empirical methods, namely:

(1) A large number of lap-joint tests are required to develop the curves of $\tau_a = f\ (L/t_1)$.

(2) A curve is only suitable for a specific set of joint-manufacturing conditions and a specific-test temperature. If, for example, the surface treatment of the adherends is altered or the final structure is to encounter a thermal environment different from that at which the tests were run new, joint strength tests must be made. This is also true to a certain extent for other tests such as the torsion cylinders although elastic properties should not be as markedly affected as strength properties would be.

(3) The method is based only on inelastic joint properties. No information is provided concerning the properties of the joint up to and just prior to failure.

(4) The method provides strength information which is probably very conservative compared to what the joint will actually withstand in a structure. The reason for this is that in a lap joint the adhesive is being subjected to a more complex-stress field with high normal-stresses present, which cause failure

at a lower level than would be expected in a structure where the joint geometry had been chosen to eliminate or minimize the peel forces.

The major advantage of the method is that lap-joint tests are simple and economical to make, and thus far it has provided a simple and workable design method.

A. Annotated Bibliography

DeBruyne, N A. (16)

The Strength of Glued Joints. Aircraft Engineering 16, pp. 115-119, April 1944.

In this paper DeBruyne introduced the concept of expressing the lap shear-strength of adhesives as a function of the square root of adherend thickness to length-of-overlap ratio. The value of this technique was shown by presenting the lap shear-strength of Redux-bonded steel, clad aluminum-alloy and aluminum metal-joints, for various combinations of adherend thickness and length of overlap. With this method all of the joint strengths were expressed on a single graph, with all the points on a single curve.

In an appendix to the paper a brief review was given of the Volkersen analysis of a supported lap joint. Using the analysis, a chart of the ratio $\tau m/\tau a$ as a function of GL^2/E_2S_2a was presented for a typical lap joint.

Broding, W. C. (11)

Criteria for Designing Adhesive-Bonded Joints. Product Engineering, 24, pp. 144-147, Oct. 1953

A general review was presented of the design of metal-bonded joints using the analysis of a restrained lap-joint where

$$\frac{\tau_m}{\tau_a} = \Delta \coth \Delta , \qquad (88)$$

and

$$\Delta^2 = \frac{GaL^2}{Et_at_1} . \qquad (89)$$

This is the same information presented in Broding (10).

Sheridan, M. L., and Merriman, H. R. (58)

Conclusions Derived from Empirical Studies of Bonded Details for Sandwich Construction. Second Pacific Area National Meeting, Los Angeles, American Society for Testing and Materials, Paper No. 83, Sept. 1956.

General conclusions were given concerning the effect of adherend-adhesive mechanical properties and geometry of the joint on the strength of various lap-type joints. The data used were collected over a period of years of testing at the Martin Company. In almost all instances the joint strengths are expressed as a function of the ratio L/t_1.

The effect of the mechanical properties of the adhesive such as modulus of elasticity, shear modulus, and shear strength on the lap-joint strength were discussed in a qualitative manner. No indication was given as to how these properties were determined.

A section was included on the development of design curves. It was pointed out that it was necessary to choose a specific L/t_1 ratio which, for a certain adhesive, would have a certain statistical probability of attaining a certain strength. It was found that an L/t_1 ratio of 25 to 30 would yield an acceptable strength 95 percent of the time. It was also found most satisfactory to design all bonded joints with a lap at least 30 times the thickness of the adherend.

Tombach, H. (61)

Predicting Strength and Dimensions of Adhesive Joints. Machine Design 29, No. 7:p. 113-120, April 1957.

A very interesting presentation was made of the various methods available for joint design. It provides as unified approach which covers the work of DeBruyne (17), Eickner (20), Volkersen (64), and Goland and Reissner (24).

It was assumed that the average failing stress of an adhesive of unit width was known, and consequently a joint of unit width had a strength (P), expressed as:

$$P = \tau_a L, \tag{90}$$

from which the strength of a joint of any width could be obtained by multiplying by the joint width.

It was assumed that (τ_a) would be a function of the parameters: L, t_a, t_1, E_a, G_a, τ_m, E_1, and μ; and from

considerations of dimensional analysis this was expressed as

$$\tau_a = \tau_m \cdot f_1 \left(\frac{L}{t_a}, \frac{L}{t_1}, \frac{E_a}{E}, \frac{G_a}{E_1}, \frac{\tau}{E_1}, \mu\right). \tag{91}$$

This relationship was simplified by making various assumptions. For example, assuming only one adhesive is available and bonded to one adherend by a specific bonding process,

$$\tau_a = \tau_m \cdot f_2 \left(\frac{L}{t_a}, \frac{L}{t_1}\right) \tag{92}$$

Assume that (S) was independent of adhesive thickness

$$\tau_a = \tau_m \cdot f_3 \left(\frac{L}{t_1}\right) \tag{93}$$

In these equations f_1, f_2, f_3 are arbitrary functions of the dimensionless parameters which must be determined in some manner either experimentally or analytically. Four approaches to determining these parameters were described including the simplifying assumptions which were necessary:

1. $$\tau_a = f\left(\frac{L}{t_1^{1/2}}\right) \tag{94}$$

The function f is determined experimentally for a large number of joints with varying overlap and adherend thickness.

2. $$\tau_a = a \left(\frac{L}{t_1}\right)^b \tag{95}$$

The constants (a) and (b) are determined experimentally by plotting (τ_a) vs. L/t_1 on log-log paper and obtaining the slope and intercept of the straight lines.

3. $$\tau_a = \frac{\tau_m}{\Delta \coth \Delta} \tag{96}$$

where $$\Delta^2 = \frac{L^2 G_a}{2 E t_a t_1} \tag{97}$$

T_m, G_a, and t_a must be determined by a technique as described by Broding (10).

4. $$\tau_a = \frac{\tau_m}{\left(\frac{1 + 3c}{4}\right) 2 \Delta \coth 2 \Delta + 3\left(\frac{1 - C}{4}\right)} \tag{98}$$

where $$\Delta^2 = \frac{L^2 G_a}{2E t_1 t_a} \tag{99}$$

$$\text{and } \frac{1}{c} = 1 + 2\sqrt{2}\tanh\left[\left(\sqrt{\frac{3}{2}\,(1-\mu^2)}\right)\left(\frac{L}{2t_1}\sqrt{\frac{P}{E_1}}\right)\right] \quad (100)$$

T_m, G_a, and t_a must be determined experimentally or by methods similar to Broding (10).

An appendix was included which gave statistical formula for obtaining the various regression lines from experimental data.

Perry, H. A. (53)

How to Calculate Stresses in Adhesive Joints. Prod. Eng. 29, pp. 64-67, July 1958.

This paper was prepared from the text "Adhesive Bonding of Reinforced Plastics" by Perry (54) and consequently all the analyses appearing in this paper are the same as in the text. Equations were given for calculating the shear stress and normal stress in the adhesive for scarf joints, butt joints, single and double lap-joints, tubular lap-joints and landed joints.

The equations for the single lap-joint were the forms of the Goland and Reissner analysis giving the maximum normal and shear stress in the adhesive, and the equations for a double lap-joint were the form of the Volkersen analysis for the maximum shear stress in the adhesive.

Anonymous (5).

Design of Honeycomb and Bonded Structures. Martin Co. Structural Design Manual ER 6131-19, pp. 30.00-30.18, 1958.

This is a portion of one of the Martin Company Structural Design Manuals. Adhesive-bonded metal-to-metal joints are classified according to three criteria:

Class 1. Best Quality - depth of lap of the joint is less than 60 times the thickness of the thinner adherend or combination of adherends.

Class 2. Intermediate Quality - depth of lap greater than 80 times the thickness of the thinner adherend.

Class 3. Nonstructural - no loads are transmitted by the adhesive, adhesive may be only a sealant.

When the design is chosen the minimum depth of lap should be 30 times the adherend thickness, but not less than 1 inch. For epoxy adhesives it should be 100 times the thinnest adherend. All joints are to be designed so that the thinnest adherend is supported by symmetry or by another rigid adherend. This is necessary to reduce any peel or tensile stresses perpendicular to the glue line.

The allowable stress in the metal at 75° F. shall be:

(1) Compression - not more than the compressive-yield stress of the metal.

(2) Shear - not more than the shear strength of the metal.

(3) Tension - not more than 95 percent of the tensile strength of the metal.

For metal gauge thicknesses longer than 0.125 inch the ends of the adherends should be tapered less than 30°.

The strength data for joints was obtained from tests of unsupported lap joints.

Brown, D. (12)

Joint Factors of Metal-Bonded Joints. Boeing Company, Wichita, Report AP-3-4C, Jan. 1960.

The purpose of this report was to verify whether the theory that joints with different geometry but the same joint factor would have the same apparent average strength. This is based on the theory of DeBruyne (16) for joints with the same adherends of equal thickness.

The theory was investigated by obtaining the lap-shear strengths for aluminum alloy and magnesium alloy bonded for varying lengths of overlap with several adhesives, and plotting these strengths as a function of

$\frac{\sqrt{t_1}}{L}$. Equations of the form

$$\tau_a = A + B\frac{\sqrt{t_1}}{L} \quad \text{were fitted to the data} \tag{101}$$

using regression analysis. It was concluded that this procedure was acceptable for lap joints with equal adherend thicknesses.

Henriksen, C. A. (31)

Molecular Engineering. Boeing Aircraft Co., Document No. D3-2867, Feb. 1960.

A portion of this report was concerned with obtaining design information on adhesive bonds. Its purpose was to provide information for a design engineer to allow him to determine the surface area required to bond a certain adherend of a given thickness with a specific adhesive, and to provide him with data concerning the strengths of various adhesive systems for certain lengths of overlap.

Graphs were constructed of lap shear strength as a function of $\sqrt{t}/L$, basing this on the Volkersen analysis of a supported lap joint. Equations of the form $y = A + B \sqrt{t}/L$ were fitted to the curves and then monograms were constructed using these equations.

Brown, D. (13)

Mechanical Properties of Structural Adhesives. Boeing Aircraft Company, Wichita, Report AP-1-22, Nov. 1960.

The purpose of this report was to determine whether the use of a joint-factor correlation method could be applied to strength of lap joints in which the adherends were alike but of different thickness, and to determine whether a joint factor other than $\frac{t}{L}$ would give better correlation with joint strength.

A large number of aluminum-alloy joints bonded with two different adhesives were tested to provide data for analysis. Lap shear strengths were plotted as a function of K, where K was determined as follows:

1. Volkersen method

$$K_1 = \frac{\sqrt{t}}{L} \quad , \quad t = \frac{t_2\ (t_1 + t_2)}{2 + 1} \quad , \quad t_1 \geq t_2 \tag{102}$$

2. $$K_2 = \frac{\sqrt{t}}{L} \quad , \quad t = \frac{t_1 + t_2}{2} \tag{103}$$

3. $$K_3 = \frac{t}{L} \quad , \quad t = \frac{t_1 + t_2}{2} \tag{104}$$

The method of least squares was used to determine the equation of the line that best fit the data. A regression analysis was made to determine whether a significant fit existed and to establish a lower limit of 99 percent confidence.

An idea of the relative fit of the lines to the data was obtained by comparing the distance of the 99 percent confidence limit line to the actual regression line. On this basis it was determined that the joint factor $K_3 = t/L$ gave the best fit of the data to a straight line.

VII. SUMMARY - PROBLEMS AND RECOMMENDATIONS FOR FURTHER RESEARCH

An important result of any survey of this type is to provide from guidance in the form of recommendations for further research. In some cases these recommendations are self evident and there may be sound reasons for their not having been undertaken in the past. In other cases the recommendations arise from a critical appraisal of the past work, and this leads to indications where work is missing or where important points need further clarification. Both types of recommendations have resulted from this review.

The recommendations for further work are given as specific research proposals or specific questions to be answered. They are divided under the three main headings used to review the literature. These proposals are presented as those representing the best judgment of the author based on the literature review and the discussions with the various company personnel visited.

As a general comment concerning the work necessary in this area, it is felt that the problems of most concern are experimental rather than analytical. There are rational analyses available for lap joints but there is still a lack of good materials data on the properties of adhesives in joints to use with these analyses. Similarly, the analyses need further experimental verification. It is generally felt that with modern computing methods and structural-analysis techniques such as matrix methods it will be possible to adequately describe joint behavior, but it is still necessary to provide materials-property data as input to any analysis. It is important for the experimentalist at this point to provide more guidance for the analyst in order that he can make more realistic assumptions concerning the deflections in the materials and their time-dependent behavior.

The other experimental point of major interest is mode of failure of adhesive joints. This question is strictly in the realm of the experimentalist and thus far has received little attention. There is always a need for knowledge of the ultimate properties of materials and any information which can be obtained concerning fracture phenomena and strength are of the greatest interest. Here again this information can guide the analyst, since this will allow him to choose the proper parameter which will adequately describe failure as a

function of applied load. It is necessary to know exactly how and by what mechanism an adhesive joint will fail.

The major concern of the analyst is to extend his work beyond the small deflection-linearized theory-of-elasticity approach to a more realistic model. With the growing interest in long-term behavior of joints and more complex thermal environments it is necessary to go beyond the first simple-elastic case. Assumptions will be necessary but they must become more realistic.

A. Properties of Adhesives in Joints.

1. Torsion Shear Tests of Adhesive Joints

Engage in an extensive test program to determine whether one of the torsion-type test specimens can be used to reproducibly determine the shear modulus of of various currently used metal-bonding adhesives.

The purpose of this testing program would be to build general confidence in the torsion-type specimen and its suitability for determining shear modulus. The greatest difficulty with this type specimen in the past has been in measuring the adhesive film-thickness which effects the computation of the shear strains and obtaining uniform cure among several specimens. The testing should be carried out to fully evalute the stress-strain-time dependent behavior of the adhesives under simple shear loading and simple tensile loading.

2. Relationship of Bulk Adhesive Properties to Within-Joint-Film Properties

Determine whether there is any fundamental different between the behavior of an adhesive existing in some bulk form or free film as compared to its behavior as a film in a joint. If there is no difference in mechanical behavior, or if the difference can be determined in some quantitative manner, it may be possible to predict joint behavior from the behavior of free films.

The objective here should be to obtain data for simple mechanical properties like shear modulus and modulus of elasticity under uniform uncomplicated-stress fields for the adhesive material both within the joint and in the free film.

A portion of this study should be to determine the effect of decreasing the adhesive-film thickness on the shear modulus and modulus of elasticity of the film in the joint.

B. Experimental Stress Analysis of Joints

1. Stress Distribution in an Adhesive Film.

Determine the three-dimensional-stress distributions in the adhesive film of actual lap-type joint rather than joint models

Past work has shown the important areas of interest are the reentrant corners of the overlap area. Further work should be undertaken to determine the effect of shape of the air-adhesive interface on the stress distribution, particularly the irregular-shaped interfaces of the type normally obtained at the ends of overlap joints in practice.

The effect of antielastic bending of the adherend sheets should be further investigated.

The stress-distribution work should be extended beyond the elastic case of material behavior up to and including fracture of the joint.

The stress distribution in the adhesive should be studied for joints of varying ratios of modulus of the adherends to modulus of adhesive. The ratio should range from 1000/1 to 10/1.

The effect on the stress distribution of decreasing the film thickness of the adhesive should be investigated.

C. Fracture Mechanics of Joints

The primary method of evaluating adhesive joints is based on determining the strength of the joint. As long as this criteria for structural failure is used it is important to obtain a better understanding of how joints fracture. This area is so new and incompletely understood, even in relation to polymers in general, that specific recommendations for further research are difficult to make. The following topics are suggested:

1. Determine how fracture surfaces are initiated in an adhesive film.

2. Determine how fracture surfaces are propagated in an adhesive film.

3. Determine how fracture initiation and propagation are affected by the various parameters of joint geometry such as adhesive-film thickness.

4. Determine how initiation and propagation are affected by the type of stress field present and rate of load.

5. Determine the effect of residual stresses on fracture mechanics.

D. Theoretical Analyses of Joints

The one important factor in analysis work is to extend the analyses beyond the assumptions of linearized small-deflection elasticity.

1. Using matrix structural-analyses techniques already developed for lap joints, extend them to include plastic and viscoelastic behavior of adhesives.

2. Using matrix-structural methods, investigate analytically the stress distributions in lap joints where the ratio of modulus of adherend to adhesive is greater than two.

APPENDIX A

General References (Annotated)

Fillunger, P. (21)

On the Strength of Soldered, Glued and Riveted Joints. Ost Wochenzeifschrift fur den offentlich Baudienst, p. 78, 1919.

It was not possible to obtain this paper for review. It was mentioned briefly by Benson (8) as an early paper discussing bonded joints, and its mention here is included only for historical interest and as a guide to those who may have access to the reference.

Mylonas, C., and DeBruyne, N. A. (50)

Static Problems. Chapt. 4; pp. 92-143 in Adhesion and Adhesives edited by N. A. DeBruyne and R. Houwink, Elsevier Publishing Co. 1951.

This paper is the first attempt by any author to provide a thorough survey of the topic of stress distributions in adhesive-bonded joints. The survey is divided into two parts: (1) Theoretical Investigation of the Stresses in Joints, and (2) Experimental Investigation of the Stresses in Joints. In each of these sections the emphasis is placed on the stress distribution in lap-type joints.

The discussion of the theoretical work is dominated by a review of work of Volkersen (64) and Goland and Reissner (24). The experimental work reviewed is primarily that of Mylonas. Short sections are included on torsion joints, butt joints, influence of the mechanical properties of adherends and adhesives, residual stresses, stresses in laminated wood, and glass to metal seals.

In the section on experimental studies some previously unpublished results are given comparing the stress distribution in a lap joint as predicted by Goland and Reissner (24) and Volkersen (64), and that obtained by an experimental stress analysis. The experimental results fell between the two predictions of which the Goland and Reissner gave the high distribution and the Volkersen the low curve. The value of the shear modulus of the adhesive, used to substitute into the analyses, was an estimated value.

Other experimental data is given on the stress distribution in a tapered-lap joint and a scarf joint similar in size to the above lap joint. These joint geometries showed only a slight reduction in the maximum stress concentration in the joint.

This review is recommended as the best summary of the earliest work in the area of mechanics of adhesive joints.

Anonymous (3)

Structural Adhesives, Lange, Maxwell, and Springer Limited, London 1951.

This is a series of lectures given on the subject, The Technology of Synthetic Resin Adhesives, by Aero Research Limited, Duxford, at Cambridge during September, 1951. The lectures ranged from the fundamentals of adhesion, chemistry of adhesives, and strength of glued joints to specific processes of gluing wood and metal.

The section on "Strength of Glued Joints" is by DeBruyne and is a simplified version of the reference by Mylonas and DeBruyne (16). Generally, the lectures are simplified and of a technical rather than scientific nature.

Perry, H. A., Jr., Hardis, L., Mathews, H. E., Jr., Briggs, L., Eagleson, E. W., and Fey, R. S. (55)

Adhesives Handbook, Part I - Engineering Principles. U.S. Naval Ordnance Laboratory, NAVORD Report 2272, Feb. 1952.

This handbook was written as an aid to the ordnance engineer using adhesives and designing adhesive-bonded structures. It includes a review of the fundamentals of adhesion, rheology of polymers, an extensive review of the paper by Goland and Reissner (24), and a review of the statistics of fracture. An actual computation of the stress distribution in a lap joint is made using the Goland and Reissner analysis. These topics are all discussed with the purpose of providing a general background of information to people unfamiliar with adhesives and adhesion.

The latter half of the Handbook covers the practical aspects of adhesive-joint design. This includes a qualitative evaluation of the mechanical efficiency of various types of joint geometry, representative strength data for various types of joints and a discussion of bonding processes. The final section covers standard inspection and test methods.

Since this is a handbook, the coverage of all the various topics is not extensive, and only an indication of the important points in each area is given.

Anonymous (4)

Structural Adhesives for Metals and Sandwich Construction. U.S. Air Force - Aircraft Industries Association Conference, Dayton, Ohio, December 1952

This conference is probably the largest ever held dealing exclusively with the properties and use of structural adhesives. At the time the conference was held, the major use of adhesives in American aircraft had been in some secondary-structural panels on the B-36 bomber and in bonded-metal helicopter-rotor blades. It is of interest that essentially nothing was said at the conference concerning design of joints from the standpoint of mechanical behavior. The papers given were concerned with development of adhesives in general, development of high-temperature adhesives, sandwich construction, bonding processes, and quality control.

Ljungström, O. (38)

Metal Bonding Practice at SAAB, (Summary), SAAB Aircraft Report KRP-0-L07, Jan. 1955.

This report contains a brief summary of various aspects of Saab Redux bonding practice, including: General policy regarding bonding application to aircraft structures, strength data, production methods, and inspection.

The general approach to the use of adhesives at SAAB was to initially use adhesive bonding on simple secondary-structural units and then as experience and confidence built up, extend the use to primary structures of complicated shape. From the design standpoint, only one adhesive (Redux) was available for use. Strength testing involved testing this adhesive under various stress conditions in different joint geometries and at various temperatures. The strength of Redux was determined in pure tension using butt joints, pure shear using a torsion specimen, and combined shear and tension in lap joints. In order to interpret the lap-joint data the lap-shear strengths are plotted as a function of the joint factor = $\sqrt{t/L}$.

Joints were tested statically and under fatigue load. Using the strength values as guides, structural panels were designed and then tested to prove the design.

Perry, H. A. (54)

Adhesive Bonding of Reinforced Plastics. Chapter 2, "Mechanics of Adhesive Joints" p. 14-42; Chapter 12, "Design of Adhesive Joints" p. 233-256; McGraw-Hill Book Company, Inc., New York, 1959.

The general format of this text by Perry follows that of his earlier Adhesives Handbook (55). The chapter on Mechanics of Adhesive Joints covers joint geometry, scarf joints, lap joints (Volkersen, Goland, and Reissner analyses), butt joints, tubular-lap joints, differential-expansion stresses, and materials behavior. The chapter on Design of Joints covers the general philosophy of joint design and discusses the design of a tubular-scarf joint, a tubular-lap joint, and tubular-tapered joint. Sections are included on design allowables, safety factors, and theories of failure. There is no information given concerning how the properties of the adhesive are to be obtained which are necessary for use in the stress analyses.

In general there are no new concepts developed in the text. It provides a good summary of the literature and serves as a good introduction to adhesives.

Benson, N. K. (8)

The Mechanics of Adhesive Bonding. Applied Mechanics Reviews 14 (2): 83-87, 1961.

This paper is an excellent and brief review of the entire field of mechanics of adhesive bonding. It summarizes the analytical aspects of lap, scarf, butt, and tubular lap joints, discusses some practical aspects of adhesives and some structural applications. The list of references is very good.

By the nature of the purpose for which this paper was written, the paper had to be relatively short, and none of the references could be reviewed in great detail. Of special interest to this review is the section on current problems of adhesive bonding which covers the points where further research is needed. These points are as follows:

(1) A better understanding of the mechanism of adhesion to better relate the theoretical field of mechanics of joints to the practical field is necessary.

(2) The physical properties of adhesive polymers are presently inadequately described in terms of a few elastic contents.

(3) Joint analyses should include the nonlinear stress-strain-time relations of the adhesives.

(4) The mechanism of failure or crack propagation in adhesives needs better description.

Sneddon, Ian (60)

The Distribution of Stress in Adhesive Joints, Chapter 9 of Adhesives, edited by D. Eley, Oxford University Press, 1961.

Sneddon presents a most current review of available literature on stress distribution and includes a listing of references that have appeared since 1951. This is the best and most comprehensive review found.

DeBruyne, N. A. (17)

The Measurement of the Strength of Adhesive and Cohesive Joints. Chapter 4: pp. 46-64 of Adhesion and Cohesion, edited by P. Weiss, Elsevier Publishing Co., 1962.

This book is a collection of the papers given at a symposium of the same title sponsored by the General Motors Research Laboratories at Warren, Mich., in July 1961. The topics discussed included adhesion of various polymers, factors affecting adhesion, strength of polymers, and techniques of measuring adhesion.

DeBruyne discusses the napkin-ring specimen for obtaining pure-shear loading of adhesives, and proposes the use of a cone and plate-type specimen, similar to that used in viscosimeters, to obtain a uniform-shear distribution. The use of the tensile shear-lap specimen as a general test for adhesives and the method of correlating the strength data from this test by using the semi-empirical joint factor $\sqrt{t/L}$ is discussed.

Tensile tests of butt joints, acceleration tests to obtain adhesive tests without any mechanical application of load, and residual stresses are briefly discussed.

APPENDIX B

Companies Visited

1. Goodyear Aircraft Co., Akron, Ohio
   Art Foerster
   Lee Beyersdorff

2. Hamilton Standard Corporation
   Division of United Aircraft
   Windsor Locks, Conn.
   Dr, Robert Cornell

3. Sikorsky Aircraft
   Division of United Aircraft
   Stratford, Conn.
   George Dmitroff
   Hugh Taylor
   Sam Lutters
   Don Neverton

4. General Dynamics Corporation, Ft. Worth, Texas
   Andrew Green

5. Bell Helicopter Co., Ft. Worth, Texas
   Neil J. MacKenzie
   M. J. McGuigan
   R. W. Metzger
   Sam Aker

6. Martin-Marietta Corporation, Orlando Div., Orlando, Florida
   George Pfaff
   Charles O'Dell
   W. E. Albert
   Harland Goplan

7. Douglas Aircraft Company, Aircraft Division, Long Beach, Calif.
   Edward Thrall
   Paul Denke
   Jim Goodwin

8. North American Aviation, Los Angeles, Calif.
   Howard Atkin
   Joseph Joanides

REFERENCES

1. Albert, W. E.
   1961. Stress Analysis of Bonded Joints. Martin Company, Orlando, Stress Analysis Memo No. 2.2,

2. Andrade, E. N. Da C.
   1911. The Distribution of Slide in a Right Six-Face Subject to Pure Shear. Proc. Royal Soc. (London) A85, pp. 448-461,

3. Anonymous
   1951. Structural Adhesives, Lange, Maxwell, and Springer Ltd., London.

4. ______ 1952. Structural Adhesives for Metals and Sandwich Construction. U.S. Air Force - Aircraft Industries Assoc. Conference, Dayton, Ohio. December.

5. ______ 1958. Design of Honeycomb and Bonded Structures. Martin Co. Structural Design Manual ER 6131-19, pp. 30.000-30.18.

6. ______ 1962. Design Study to Improve the Structural Efficiency of Polaris Filament Wound Motor Cases. Goodyear Aircraft Corp., GER 10741.

7. Arnovlević, I.
   1909. Das Verteilungsgesetz der Haftspannung bei axial bean-spruchten Verbundstäben. Zeitschrift für Architektur und Ingenierwesen, Hannover, 2:414-418.

8. Benson, N. K.
   1961. The Mechanics of Adhesive Bonding. Applied Mechanics Reviews, 14(2):83-87.

9. Broding, W. C.
   1952. Analysis for Mechanical and Bonded Joints in Metalite. Chance Vought Aircraft Engineering Dept. Report No. 7588.

10. Broding, W. C.
1952. Determination of Static Shear Strength of Redux for Design. Chance Vought Aircraft Report No. 7575.

11. ______
1953. Criteria for Designing Adhesive-Bonded Joints. Product Eng. 24, pp. 144-147.

12. Brown, D.
1960. Joint Factors of Metal-Bonded Joints. Boeing Aircraft Co., Wichita, Report AP-3-4C.

13. ______
1960. Mechanical Properties of Structural Adhesives. Boeing Aircraft Co., Wichita, Report AP-1-22.

14. Coker, E. G.
1912. An Optical Determination of the Variation of Stress in a Thin Rectangular Plate Subjected to Shear. Proceedings Royal Society (London), 86(A587):291-319.

15. Cornell, R. W.
1953. Determination of Stresses in Cemented Lap Joints. J. Appl. Mech. 20, No. 3, pp. 355-364.

16. DeBruyne, N. A.
1944. The Strength of Glued Joints. Aircraft Engineering 16, p. 115-118.

17. ______
1962. The Measurement of the Strength of Adhesive and Cohesive Joints. Chapter 4, pp. 46-64 of Adhesion and Cohesion edited by P. Weiss, Elsevier Publishing Co.

18. Demarkles, L. R.
1955. Investigation of the Use of a Rubber Analog in the Study of Stress Distribution in Riveted and Cemented Joints. NACA Tech. Note 3413.

19. Dietz, A. G. H., Grinsfelder, H., and Reissner, E.
1946. Glue Line Stresses in Laminated Wood. ASME Transactions 68, p. 329-335.

20. Eickner, H.
1955. Basic Shear Strength Properties of Metal-Bonding Adhesives as Determined by Lap-Joint Stress Formulas of Volkersen and Goland and Reissner. Forest Products Lab. Rept. No. 1850.

21. Fillunger, P.
1919. On the Strength of Soldered, Glued and Riveted Joints. Ost Wochenzeitschrift fur den Offentlich Baudienst, p. 78.

22. Filon, L. N. G.
1903. IV. On an Approximate Solution for the Bending of a Beam of Rectangular Cross-Section Under Any System of Load, with Special Reference to Points of Concentrated or Discontinuous Loading. Part III. Solution for a Beam Whose Upper and Lower Boundaries are Acted Upon by Shearing Stress Only. Roy. Soc. London, Philos. Trans. A 201, p. 63-129.

23. Gillespie, T., and Rideal, E.
1956. The Deformation and Strength of Napkin Ring Metal-Adhesive-Metal Joints. J. Coll. Sci. 11, pp. 732-747.

24. Goland, M., and Reissner, E.
1944. Stresses in Cemented Joints. Journal Applied Mechanics, 11(1):A17-A27.

25. Goldenburg, D.
1948. Distribution of Shear Stresses in Bonded Joints. Chance Vought Aircraft Report No. 7441.

26. Goodwin, J. F.
1962. Research and Thermomechanical Analysis of Brazed or Bonded Structural Joints. Douglas Aircraft Report on Contract AF 33(657)8542.

27. Hahn, K. F.
1960. Photostress Investigation of Bonded Lap Joints; Part II. Analysis of Experimental Data. Douglas-Aircraft Company Research Report SM 4000-1.

28. ______
1960. Stress Distribution in the Adherends of Bonded Lap Joints. Douglas Aircraft Research Report SM 40010.

29. ______
1961. Lap Shear and Creep Performance in Metal-to-Metal Joints. Adhesives Age 4, No. 12, pp. 34-36.

30. Hahn, K. F., and Fouser, D. F.
1962. Methods of Determining Stress Distribution in Adherends and Adhesives. Symposium on Structural Adhesives, Picatinny Arsenal, 1961, J. Appl. Poly. Sci. 6, p. 145-149.

31. Henrikson, C. A.
1960. Molecular Engineering. Boeing Aircraft Co., Document No. D3-2867.

32. Inglis, C. E.
1923. Stress Distribution in a Rectangular Plate Having Two Opposing Edges Sheared in Opposite Directions. Proc. Royal Soc. (London) A 103, pp. 598-610.

33. Ito, K.
1960. Note on Adhesive Strength Test by Lap Joints. Inst. Phys. Chem. Res. Tokyo, Scientific Paper 54, pp. 295-306.

34. Jackson, C. C.
1943. Joints - Lap, Efficiency, Graphical Study of Physical Factors. Chrysler Corporation Engineering. Report No. 4904.

35. Kuenzi, E. W.
1956. Determination of Mechanical Properties of Adhesives for Use in the Design of Bonded Joints. Forest Products Lab. Report No. 1851.

36. ______, and Stevens, G. H.
1963. Determination of Mechanical Properties of Adhesives for Use in the Design of Bonded Joints. U.S. Forest Service Res. Note FPL-011.

37. Kutscha, D.
1962. Photoelastic Analysis of Shear Stress Distribution in Adhesive-Bonded Lap Joints. Forest Prod. Lab. Report TP-122.

38. Ljungstrom, O.
1955. Metal Bonding Practice at Saab (Summary). SAAB Aircraft Report KRP-0-L07.

39. Lobbett, J. W., and Robb, E. A.
1962. Thermo-Mechanical Analysis of Structural Joint Study. WADD TR 61-151.

40. Love, A. E. H.
1927. Mathematical Theory of Elasticity, Chapter IX, Cambridge University Press.

41. Lubkin, J. L.
1953. A Theory of Adhesive Scarf Joints, with Analysis of Test Results. Phase I Report, U.S. Naval Ordnance Contract NOrd 13383.

42. ______
1954. The Stress Distribution in Adhesive Joints. Midwest Research Institute Technical Report, Contract NOrd 13383.

43. Lunsford, L. R.
1960. Bonded Metal-to-Metal Shear Testing. ASTM Sp. Tech. Publ. 289.

44. ______
1961. Stress Distribution in Bonded Joints. General Dynamics, Ft. Worth ERR-FW-130, Structures.

45. ______
1962. Adhesive Torsional-Shear Test. General Dynamics - Ft. Worth, Report ERR-FW-134 (Structures).

46. McLaren, A. S., and MacInnes, J.
1958. The Influence on the Stress Distribution in an Adhesive Lap Joint of Bending of the Adhering Sheets. British Journal of Applied Physics, 9:72-77.

47. Misztal, I.
1956. Stress Distribution in Glued Single and Multijoints of Sheets Subjected to Shear Along the Splice Line. Akademija Uniejetnosci, Krakow Bull Akad. Polonaise Sci. Cl., IV 4, pp. 21-27.

48. Mylonas, C.
1948. On the Stress Distribution in Glued Joints. Proceedings 8th International Congress of Applied Mechanics, London, 137-149.

49. ______
1954. Experiments on Composite Models with Application to Cemented Joints. Proceedings Society Experimental Stress Analysis, 12:129-142.

50. Mylonas, C., and DeBruyne, N. A.
1951. Static Problems. Chapter 4: pp. 92-143 in Adhesion and Adhesives edited by N. A. DeBruyne and R. Houwink, Elsevier Publishing Co.

51. Niskanen, E.
1955. On the Distribution of Shear Stress in a Glued Specimen of Isotropic or Anisotropic Material. The State Institute for Technical Research, Finland. Jul. Kaisu 30 Publication.

52. Norris, C. B., and Ringelstetter, L. A.
1950. Shear Stress Distribution Along Glue Line Between Skin and Cap-Strip of an Aircraft Wing. NACA TN 2152.

53. Perry, H. A.
1958. How to Calculate Stresses in Adhesive Joints. Prod. Eng. 29, p. 64-67.

54. _______
1959. Adhesive Bonding of Reinforced Plastics, Chapter 2, Mechanics of Adhesive Joints. McGraw Hill Co., Inc., New York,

55. _______, Hardis, L., Mathews, H. E., Briggs, L., Eagleson, E. W. and Fey, R. S.
1952. Adhesives Handbook, Part I, Engineering Principles. U.S. Naval Ordnance Laboratory, NAVORD Report 2272.

56. Plantema, F. J.
1947. De Schuifspanning in een Lijmraad. Nat. Luchtvaart Laboratorium, Amsterdam, Report MI 181.

57. Seely, F. B., and Smith, J. O.
1952. Advanced Mechanics of Materials. John Wiley and Sons.

58. Sheridan, M. L., and Merriman, H. R.
1956. Conclusions Derived from Empirical Studies of Bonded Details for Sandwich Construction. Second Pacific Area National Meeting, Los Angeles. ASTM Paper No. 83, Sept. 1956.

59. Sherrer, R. E.
1957. Stresses in a Lap Joint with Elastic Adhesive. USDA Forest Products Lab. Report No. 1864.

60. Sneddon, I.
1961. The Distribution of Stress in Adhesive Joints, Chapter 9 of Adhesives, edited by D. Eley, Oxford Univ. Press,